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Sudakov resummation for prompt-photon production in hadron collisions*

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Abstract: We present the explicit expressions for the resummation of large-x_T Sudakov effects in the transverse-energy distribution of prompt-photons produced in hadronic collisions, to next-to-leading logarithmic (NLL) accuracy. Fragmentation processes do not contribute to the Sudakov resummation at NLL level. In Mellin space, the resummed radiative factor is the product of independent radiators for each external coloured parton appearing in the Born process, times a simple factor describing the interferences between initial- and final-state soft-gluon radiation. The formulae are given in terms of Mellin moments, and can be used for phenomenological applications using standard techniques for the inverse-Mellin transforms. The calculations presented in this work, when added to the existing works on DY and DIS production, complete the theoretical ground-work necessary to carry out global fits of parton densities with a uniform NLL accuracy in the large-x region.

Keywords: QCD, NLO Computations, Hadronic Colliders.

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Contents

1. Introduction

Prompt-photon production plays a very important role in our understanding of the physics of hadron collisions. At the leading order (LO) in QCD perturbation theory, prompt photons are produced via light-quark annihilation, with emission of a hard gluon recoiling against the photon, or via quark-gluon Compton scattering, with the emission of a quark. When the Compton process dominates the cross section, tests or even measurements of the gluon density inside the proton can be performed. This is the case of photon production at small $x_T = 2E_T/\sqrt{S}$ ($x_T \lesssim 0.1$) in $p\bar{p}$ collisions, and it is true for all values of x_T that are accessible in fixed-target proton-nucleon collisions, due to smaller content of antiquarks relative to gluons in the nucleon sea. In particular, prompt-photon production at large x_T can therefore be used to constrain or measure the gluon density at large x. This region of the gluon density is of great importance for the study of high-transverse-momentum phenomena at hadron colliders, and it is not accessible using only Deep-Inelastic-Scattering (DIS) data.

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The cross section for inclusive photon production has been computed at the next-to-leading order (NLO) in perturbation theory [1, 2, 3]. The NLO computation of *isolated*-photon production [4, 5, 6, 7], which is the relevant quantity for the measurements carried out in high-energy $p\bar{p}$ collisions, is also available in the limit of small size of the isolation cone [8, 9]. These calculations include all light-parton fragmentation processes up to NLO [10].

These theoretical results have been used [11, 12, 13, 14] to probe the overall consistency of the prompt-photon production data from both fixed-target [15, 16] and collider experiments [17, 18, 19].

The interpretation of the data has not provided so far a fully satisfactory picture. The study by Huston et al. [13] exposed a tendency of the x_T distributions to be steeper than theory, regardless of the value of $\sqrt{S}$. This result could not be accomodated by a simple modification of the parton densities, since different experiments probe different values of x_T. These authors therefore proposed that additional mechanisms should be introduced to explain the pattern of the data. In the fixed-target regime, such a mechanism would be provided by the presence of a non-perturbative k_T kick, which would smear the E_T spectra. This phenomenon was also advocated to help explain the spectra of fixed-target heavy-quark production [20]. In the high-energy regime, probed by the Tevatron experiments, the E_T smearing necessary to reconcile theory and data could be provided by the inclusion of multi-gluon emission effects from the evolution of the initial state, as advocated by Baer and Reno [21].

The analysis by Vogelsang and Vogt [14] indicated that allowing for different choices of factorization and renormalization scales, a satisfactory fit to the data could be accomodated by modifying the gluon density within the range allowed by the DIS data available in 1995. This interpretation is apparently not viable anymore [22], because of the most recent constraints on the gluon density extracted at small x from the HERA data, and, in particular, because of the latest fixed-target prompt-photon data from E706 [23].

The comparison of the E706 data with NLO QCD, carried out in [23], seems to confirm the need for an intrinsic-k_T smearing corresponding to $\langle k_T \rangle \sim 1$ GeV. These conclusions are shared in a recent global fit of the parton densities performed by the MRST group [24]. In this same study (see also [25]), however, a strong dependence of $\langle k_T \rangle$ on $\sqrt{S}$ is claimed to be necessary to properly describe the lower-energy data published by WA70.

In conclusion, the comparison of the large-x_T fixed-target data with NLO QCD still presents some puzzling features, which will need to be properly clarified before use of these data can be made to place robust constraints on the large-x gluon distribution inside the proton. This is unfortunate, since these data provide today the only independent probe on high-x gluons. Their accurate interpretation is therefore a fundamental ingredient for an accurate prediction of the production rate of high-E_T jets at the Tevatron, a measurement which has challenged perturbative QCD in the recent past [26].

To improve the reliability of the perturbative predictions for the production of prompt photons at large x_T, and detect the presence of potentially large corrections beyond NLO that could change the interpretation of the current data, in the present work we consider an extension of the NLO formalism that includes large logarithmically-enhanced effects as the production threshold is approached. This is the kinematical region of interest for the fixed-target data. As the x_T of the photon is increased, the parton luminosity becomes steeper, being driven down by the strong suppression of the gluon density at large x. We thus enter a regime of inhibited radiation: further radiation of soft gluons is strongly suppressed, and logarithmically-enhanced effects (Sudakov effects) arise at any order in the perturbative expansion. These logarithms spoil the reliability of the fixed-order expansion in the strong coupling α_s and, hence, their summation to all pertubative orders is necessary. For simplicity, this regime can be described in terms of the distance from the kinematic threshold, which is reached when $x_T \sim 1$. In this limit, the coefficients of the perturbative series for the cross section are enhanced by powers of $\ln(1 - x_T)$ that have to be resummed at all orders. This simplified description applies to the case of hypothetical structure functions that are not strongly suppressed at high x. It is, however, an appropriate framework for the classification of the perturbative corrections we are interested in.

In this work we will present all the formalism that is needed to compute the resummed cross section for direct photons, integrated over the photon rapidity and at fixed transverse energy. In particular, we give explicit resummation formulae that are valid up to next-to-leading logarithmic (NLL) accuracy. No phenomenological applications will be discussed here, but they will be explored in a forthcoming work. Furthermore, the formulae for the resummed correction factors will be presented and illustrated, but not derived here. The general formalism [27] used to obtain the resummation factors, which has already been used for the NLL resummation of the heavy-quark total production cross-section in ref. [28], will be presented in a forthcoming publication [29].

The rest of this work is organised as follows. The general theoretical framework is discussed in sect. 2. In sect. 3 we fix our notation and present the formulae for the Born cross section, together with their Mellin transforms. Soft-gluon resummation at large x_T is considered in sect. 4. The NLL resummation factors are presented in subsects. 4.1, 4.2. In subsect. 4.3, the fixed-order expansion of the resummed formulae is compared with the NLO results of refs. [1, 3]. This comparison is also exploited to fix certain constant factors in the resummed formulae. In sect. 5 we discuss similarities and differences between the resummed factors for the prompt-photon cross section and those for other hard-scattering processes, and we prove the consistency of the results obtained in the case of prompt photons with the coherence properties of large-angle soft-gluon emission. Section 6 contains our conclusions.

More technical details are left to the Appendices. In Appendix A, we give the NLL formulae for the radiative factors in the Mellin transform representation. Previous experience in the case of heavy-flavour production has shown that this is what is needed

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to perform a reliable phenomenological analysis [28]. In Appendix B, the threshold limit of the partonic cross sections is discussed. In particular, a prediction for the logarithmic terms at the next-to-next-to-leading order (NNLO) is given. Finally, in Appendix C the simpler case of photoproduction of direct photons is discussed.

While completing this paper, a study of the NLL resummation for single-inclusive distributions, covering the case of prompt-photon production, has been released by Laenen, Oderda and Sterman [30].

2. General framework

The presence of large logarithmically-enhanced contributions is a common feature in the study of the production cross sections of systems of high mass or high transverse energy near threshold. In this kinematic regime, known as the Sudakov regime, only additional soft gluons can be produced. The radiative tail of the real emission is thus strongly suppressed and cannot balance the virtual corrections. The imperfect compensation between real and virtual terms leads to the large logarithmic contributions.

General techniques for resumming soft-gluon corrections to hadroproduction processes have been developed over the past several years, starting from the case of Drell-Yan (DY) pair production [31, 32]. The resummation program of the soft-gluon contributions is best carried out in the Mellin-transform space, or N-space, where N denotes the parameter that is conjugate to the kinematic variable that measures the distance from threshold. In N-moment space the threshold-production region corresponds to the limit $N \to \infty$ and the typical structure of the logarithmic contributions is as follows

$$\hat{\sigma}_N^{(0)} \left\{ 1 + \sum_{n=1}^{\infty} \alpha_s^n \sum_{m=1}^{2n} c_{n,m} \ln^m N \right\} , \tag{2.1}$$

where $\hat{\sigma}_N^{(0)}$ is the corresponding partonic cross-section at LO. In the DY process the logarithmic terms in the curly bracket of eq. (2.1) can be explicitly summed and organized in a radiative factor $\Delta_{DY,N}$ that has an exponential form [31, 32, 33]:

$$\begin{aligned} \Delta_{DY,N}(\alpha_s) &= \exp \left\{ \sum_{n=1}^{\infty} \alpha_s^n \sum_{m=1}^{n+1} G_{nm} \ln^m N \right\} & (2.2) \\ &= \exp \left\{ \ln N \, g_{DY}^{(1)}(\alpha_s \ln N) + g_{DY}^{(2)}(\alpha_s \ln N) + \alpha_s g_{DY}^{(3)}(\alpha_s \ln N) + \ldots \right\} . & (2.3) \end{aligned}$$

Note that the exponentiation in eq. (2.2) is not trivial. The sum over m in eq. (2.1) extends up to $m = 2n$ while in eq. (2.2) the maximum value for m is smaller, $m \leq n+1$. In particular, this means that all the double logarithmic (DL) terms $\alpha_s^n c_{n,2n} \ln^{2n} N$ in eq. (2.1) are taken into account by simply exponentiating the lowest-order contribution $\alpha_s c_{1,2} \ln^2 N$. Then, the exponentiation in eq. (2.2) allows one to define the improved perturbative expansion in eq. (2.3). The function $\ln N \, g_{DY}^{(1)}$ resums all the *leading* logarithmic (LL) contributions $\alpha_s^n \ln^{n+1} N$, $g_{DY}^{(2)}$ contains the *next-to-leading* logarithmic (NLL) terms $\alpha_s^n \ln^n N$, $\alpha_s g_{DY}^{(3)}$ contains the *next-to-next-to-leading* logarithmic (NNLL) terms $\alpha_s^{n+1} \ln^n N$, and so forth. Once the functions $g_{DY}^{(k)}$ have been computed, one has a systematic perturbative treatment of the region of N in which $\alpha_s \ln N \lesssim 1$, which is much larger than the domain $\alpha_s \ln^2 N \ll 1$ in which the fixed-order calculation in α_s is reliable.

The QCD exponentiation formula for the DY process formally resembles analogous results for QED. This is because the underlying hard-scattering subprocess involves only *two* QCD partons, i.e. the annihilating $q\bar{q}$ pair, and, hence, both its kinematics and its colour structure are simple. In the case of prompt-photon production, instead, all the LO hard-scattering subprocesses

$$q + \bar{q} \to g + \gamma \,, \quad q + g \to q + \gamma \,, \quad \bar{q} + g \to \bar{q} + \gamma \,, \tag{2.4}$$

involve *three* coloured partons and, then, Sudakov resummation is by far less trivial.

A key ingredient for the exponentiation in the DY process is the factorization of the corresponding multigluon matrix elements in the soft limit. Since the colour structure of a two-parton hard-scattering is trivial[1], primary soft radiation from the two hard partons factorizes as in QED. Then the subsequent parton radiation can be factorized in non-interfering angular-ordered cascades because of the coherence properties [35] of QCD emission.

In the case of prompt-photon production, and, in general, in scattering processes produced by hard interactions of more than two QCD partons, the colour and momentum flows in the partonic subprocess are more involved. In particular, the interplay between colour exchange in the hard scattering and colour transitions induced by parton radiation spoils QED-like factorization of soft-gluon emission. Therefore, both *colour correlations* and *soft-gluon interferences* have to be properly taken into account. It turns out that, in general, the threshold logarithmic corrections cannot be resummed in a single exponential factor [36]: one has to deal with exponential matrices that couple the various colour channels of the hard-scattering subprocess.

However, the three-parton subprocesses in eq. (2.4) are a special case among the multiparton configurations. There is only *one colour-singlet* state[2] that can be constructed by combining $q\bar{q}g$ and then, because of colour conservation, soft-gluon radiation cannot induce colour transitions in the hard-scattering subprocess. Owing to the absence of colour correlations, we conclude that the logarithmically-enhanced threshold corrections in prompt-photon hadroproduction are embodied by three radiative factors (one factor for each of the LO partonic channels in eq. (2.4))

$$\Delta_N^{q\bar{q}\to g\gamma}(\alpha_s) \,, \quad \Delta_N^{qg\to q\gamma}(\alpha_s) \,, \quad \Delta_N^{\bar{q}g\to \bar{q}\gamma}(\alpha_s) \,, \tag{2.5}$$

that, after all-order resummation, have an exponential form analogous to the DY radiative factor in eq. (2.3). Nonetheless, the similarity with the DY process regards only

[1]This is the reason why similar exponentiation formulae apply to many other *two-jet*-dominated processes [34].

[2]In other words, the $q\bar{q}g$ colour-amplitude $\mathcal{M}_{q\bar{q}g}^{\alpha\bar{\alpha}a}$ ($\alpha, \bar{\alpha}$ and a are the colour indices of the quark, antiquark and gluon, respectively) is necessarily proportional to the matrix $t^a_{\alpha\bar{\alpha}}$ of the fundamental representation of the gauge group $SU(N_c)$.

the colour structure. The hard-scattering kinematics is different in prompt-photon production and the factors in eq. (2.5) still contain soft-gluon *interference effects* that are non-trivial. The pattern of these soft-gluon interferences is typical of multiparton hard-scatterings.

General theoretical methods to perform Sudakov resummation in processes initiated by hard scattering of more than two QCD partons have recently been developed by two groups. The KOS formalism [36, 37, 38] uses the Wilson line approach to treat colour correlations and soft-gluon interferences. It has been explicitly applied to the calculation with NLL accuracy of the the invariant-mass distributions of heavy-quark pairs and dijets (see also ref. [30]). The more recent BCMN formalism [28] is based on generalized soft-gluon factorization and has been used for the NLL calculation of the total cross section for heavy-quark hadroproduction. The consistency of the NLL results for the total cross section [28] with those for the invariant mass distribution [36] of heavy-quark pairs shows that, although different, the two formalisms are equivalent to a large extent.

In the rest of this paper, we first introduce our notation and then we present the resummed expressions of the prompt-photon radiative factors (2.5) to NLL accuracy. The results include the complete soft-gluon interferences to this accuracy, as evaluated by using the BCMN formalism. Details of our general formalism will be presented elsewhere [29].

3. Notation and fixed-order calculations

We consider the inclusive production of a single prompt photon in hadron collisions:

$$H_1(P_1) + H_2(P_2) \to \gamma(p) + X \, . \tag{3.1}$$

The colliding hadrons H_1 and H_2 respectively carry momenta P_1^ν and P_2^ν. In their centre-of-mass frame, using massless kinematics, they have the following light-cone coordinates

$$P_1^\nu = \sqrt{\frac{S}{2}}\,(1, \mathbf{0}, 0) \, , \qquad P_2^\nu = \sqrt{\frac{S}{2}}\,(0, \mathbf{0}, 1) \, , \tag{3.2}$$

where $S = (P_1 + P_2)^2$ is the centre-of-mass energy squared. The photon momentum p is thus parametrized as

$$p^\nu = \left(\frac{E_T}{\sqrt{2}}\, e^{\eta}, \mathbf{E}_T, \frac{E_T}{\sqrt{2}}\, e^{-\eta} \right) , \tag{3.3}$$

where E_T and η are the transverse energy and the pseudorapidity, respectively. We also introduce the customary scaling variable x_T $(0 \le x_T \le 1)$:

$$x_T = 2 \frac{E_T}{\sqrt{S}} \, . \tag{3.4}$$

We are interested in the prompt-photon production cross section integrated over η at fixed E_T. According to perturbative QCD, the cross section is given by the following

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factorization formula

$$\frac{d\sigma_\gamma(x_T,E_T)}{dE_T} = \frac{1}{E_T^3}\sum_{a,b}\int_0^1 dx_1\, f_{a/H_1}(x_1,\mu_F^2)\int_0^1 dx_2\, f_{b/H_2}(x_2,\mu_F^2)\times$$
$$\times\int_0^1 dx\left\{\delta\left(x-\frac{x_T}{\sqrt{x_1x_2}}\right)\hat\sigma_{ab\to\gamma}(x,\alpha_s(\mu^2);E_T^2,\mu^2,\mu_F^2,\mu_f^2)+\right.$$
$$\left.+\sum_c\int_0^1 dz\, z^2\, d_{c/\gamma}(z,\mu_f^2)\,\delta\left(x-\frac{x_T}{z\sqrt{x_1x_2}}\right)\hat\sigma_{ab\to c}(x,\alpha_s(\mu^2);E_T^2,\mu^2,\mu_F^2,\mu_f^2)\right\}, \quad (3.5)$$

where a, b, c denote the parton indices ($a = q, \bar q, g$), and $f_{a/H_1}(x_1,\mu_F^2)$ and $f_{b/H_2}(x_1,\mu_F^2)$ are the parton densities of the colliding hadrons evaluated at the factorization scale μ_F. The first and the second term in the curly bracket on the right-hand side of eq. (3.5) respectively represent the *direct* and the *fragmentation* component of the cross section. The fragmentation component involves the parton fragmentation function $d_{c/\gamma}(z,\mu_f^2)$ of the observed photon at the factorization scale μ_f, which, in general, differs from the scale μ_F of the parton densities.

The *rescaled*[3] partonic cross sections $\hat\sigma_{ab\to\gamma}$ and $\hat\sigma_{ab\to c}$ in eq. (3.5) are computable in QCD perturbation theory as power series expansions in the running coupling $\alpha_s(\mu^2)$, μ being the renormalization scale in the $\overline{\rm MS}$ renormalization scheme:

$$\hat\sigma_{ab\to\gamma}(x,\alpha_s(\mu^2);E_T^2,\mu^2,\mu_F^2,\mu_f^2) =$$
$$\alpha\,\alpha_s(\mu^2)\left[\hat\sigma^{(0)}_{ab\to d\gamma}(x)+\sum_{n=1}^{\infty}\alpha_s^n(\mu^2)\,\hat\sigma^{(n)}_{ab\to\gamma}(x;E_T^2,\mu^2,\mu_F^2,\mu_f^2)\right], \quad (3.6)$$

$$\hat\sigma_{ab\to c}(x,\alpha_s(\mu^2);E_T^2,\mu^2,\mu_F^2,\mu_f^2) =$$
$$\alpha_s^2(\mu^2)\left[\hat\sigma^{(0)}_{ab\to c}(x)+\sum_{n=1}^{\infty}\alpha_s^n(\mu^2)\,\hat\sigma^{(n)}_{ab\to c}(x;E_T^2,\mu^2,\mu_F^2,\mu_f^2)\right]. \quad (3.7)$$

Note that the ratio between the direct and the fragmentation terms in eqs. (3.6) and (3.7) is of the order of α/α_s, where α is the fine structure constant. This ratio is compensated by the photon-fragmentation function $d_{c/\gamma}$, which (at least formally) is of the order of α/α_s, so that direct and fragmentation components equally contribute to eq. (3.5).

Throughout the paper we always use parton densities and parton fragmentation functions as defined in the $\overline{\rm MS}$ factorization scheme. In general, we consider different values for the renormalization and factorization scales μ, μ_F, μ_f, although we always assume that all of them are of the order of the photon transverse energy E_T.

The LO terms $\hat\sigma^{(0)}_{ab\to d\gamma}$ in eq. (3.6) are due to the tree-level parton scatterings

$$a+b\to d+\gamma\,, \quad (3.8)$$

[3]These functions are related to the partonic differential cross sections by $\hat\sigma_{ab\to i} = E_T^3 d\hat\sigma_{ab\to i}/dE_T$ ($i=\gamma,c$).

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where the flavour indices a, b, d are those explicitly denoted in the subprocesses of eq. (2.4). Using our normalization, the two independent (non-vanishing) partonic cross sections for the direct component are:

$$\hat{\sigma}^{(0)}_{q\bar{q}\to g\gamma}(x) = \pi\, e_q^2\, \frac{C_F}{N_c}\, \frac{x^2}{\sqrt{1-x^2}} \left(2 - x^2\right) \tag{3.9}$$

$$\hat{\sigma}^{(0)}_{qg\to q\gamma}(x) = \hat{\sigma}^{(0)}_{\bar{q}g\to \bar{q}\gamma}(x) = \pi\, e_q^2\, \frac{1}{2N_c}\, \frac{x^2}{\sqrt{1-x^2}} \left(1 + \frac{x^2}{4}\right), \tag{3.10}$$

where e_q is the quark electric charge. Note that, having integrated over the photon pseudorapidity, the expressions (3.9, 3.10) are even functions of the photon transverse energy E_T, i.e. they depend on x^2 rather than on x. The NLO terms $\hat{\sigma}^{(1)}_{ab\to\gamma}$ in eq. (3.6) were first computed in ref. [1].

The partonic contributions $\hat{\sigma}_{ab\to c}$ to the fragmentation component of the cross section are exactly equal to those of the single-hadron inclusive distribution. Their explicit calculation up to NLO was performed in ref. [10].

We are mainly interested in the behaviour of QCD corrections near the partonic-threshold region $x \to 1$, i.e. when the transverse energy E_T of the photon approaches the partonic centre-of-mass energy $\sqrt{x_1 x_2 S}$. In this region, the LO cross sections (3.9), (3.10) behave as

$$\hat{\sigma}^{(0)}_{ab\to d\gamma}(x) \sim \frac{1}{\sqrt{1-x^2}}\,. \tag{3.11}$$

This integrable singularity is a typical phase-space effect. At higher perturbative orders, the singularity in eq. (3.11) is enhanced by double-logarithmic corrections due to soft-gluon radiation and the cross section contributions in eqs. (3.6), (3.7) behave as

$$\hat{\sigma}^{(n)}(x) \sim \hat{\sigma}^{(0)}(x)\, \ln^{2n}(1-x)\,. \tag{3.12}$$

Resummation of these soft-gluon effects to all orders in perturbation theory can be important to improve the reliability of the QCD predictions.

3.1. N-moment space

The resummation program of soft-gluon contributions has to be carried out [31, 32] in the Mellin-transform space, or N-space. Working in N-space, one can disentangle the soft-gluon effects in the parton densities from those in the partonic cross section and one can straightforwardly implement and factorize the kinematic constraints of energy and longitudinal-momentum conservation.

It is convenient to consider the Mellin transform $\sigma_{\gamma,N}(E_T)$ of the dimensionless hadronic distribution $E_T^3 d\sigma_\gamma(x_T, E_T)/dE_T$. The N-moments with respect to x_T^2 and at fixed E_T are thus defined as follows:

$$\sigma_{\gamma,N}(E_T) \equiv \int_0^1 dx_T^2\, (x_T^2)^{N-1}\, E_T^3 \frac{d\sigma_\gamma(x_T, E_T)}{dE_T}\,. \tag{3.13}$$

In N-moment space, eq. (3.5) takes a simple factorized form

$$\sigma_{\gamma,\,N}(E_T) = \sum_{a,b} f_{a/H_1,\,N+1}(\mu_F^2)\, f_{b/H_2,\,N+1}(\mu_F^2) \times$$
$$\times\Big\{ \hat\sigma_{ab\to\gamma,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2) +$$
$$+ \sum_c \hat\sigma_{ab\to c,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2)\, d_{c/\gamma,2N+3}(\mu_f^2) \Big\}\,, \tag{3.14}$$

where we have introduced the customary N-moments $f_{a/H,\,N}$ and $d_{a/\gamma,\,N}$ of the parton densities and parton fragmentation functions:

$$f_{a/H,\,N}(\mu^2) \equiv \int_0^1 dx\, x^{N-1}\, f_{a/H}(x, \mu^2)\,, \tag{3.15}$$
$$d_{a/\gamma,\,N}(\mu^2) \equiv \int_0^1 dz\, z^{N-1}\, d_{a/\gamma}(z, \mu^2)\,. \tag{3.16}$$

Note that the N-moments of the partonic cross sections in eq. (3.14) are again defined with respect to x_T^2:

$$\hat\sigma_{ab\to\gamma,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2)$$
$$\equiv \int_0^1 dx^2\, (x^2)^{N-1}\, \hat\sigma_{ab\to\gamma}(x, \alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2)\,. \tag{3.17}$$

In particular, the N-moments of the LO contributions in eqs. (3.9), (3.10) are given by the following explicit expressions:

$$\hat\sigma^{(0)}_{q\bar q\to g\gamma,\,N} = \pi\, e_q^2\, \frac{C_F}{N_c}\, \frac{\Gamma(1/2)\,\Gamma(N+1)}{\Gamma(N+5/2)}\, (2+N)\,, \tag{3.18}$$

$$\hat\sigma^{(0)}_{qg\to q\gamma,\,N} = \hat\sigma^{(0)}_{\bar q g\to \bar q\gamma,\,N} = \pi\, e_q^2\, \frac{1}{8N_c}\, \frac{\Gamma(1/2)\,\Gamma(N+1)}{\Gamma(N+5/2)}\, (7+5N)\,. \tag{3.19}$$

Note also the pattern of moment indices in the various factors of eq. (3.14), i.e. $f_{a/H,\,N+1}$ for the parton densities and $d_{c/\gamma,2N+3}$ for the parton fragmentation functions. This non-trivial pattern follows from the conservation of the longitudinal and transverse momenta.

The threshold region $x_T \to 1$ corresponds to the limit $N \to \infty$ in N-moment space. In this limit, the soft-gluon corrections (3.12) to the higher-order contributions of the partonic cross sections become

$$\hat\sigma_N^{(n)} \sim \hat\sigma_N^{(0)}\, \ln^{2n} N\,. \tag{3.20}$$

The resummation of the soft-gluon logarithmic corrections to all orders in perturbation theory is considered in the following Section.

4. Soft-gluon resummation at high E_T

4.1. Resummed cross section to NLL accuracy

In the threshold or large-N limit, the various partonic channels contribute in different ways to the prompt-photon cross section $\sigma_{\gamma,\,N}(E_T)$ in eq. (3.14).

Firstly, we can compare the direct and fragmentation contributions to eq. (3.14). The partonic cross sections $\hat{\sigma}_{ab\to\gamma,\,N}$ and $\hat{\sigma}_{ab\to c,\,N}$ have the same large-N behaviour, but, owing to the hard (although collinear) emission always involved in any splitting process $c\to\gamma+X$, the photon-fragmentation function $d_{c/\gamma,\,N}$ is of the order of $1/N$. Therefore, in eq. (3.14) the fragmentation component is formally suppressed by a factor of $1/N$ with respect to the direct component and in our resummed calculation we can neglect the fragmentation contributions.

Then, we can discuss the differences in the large-N behaviour of the partonic cross sections $\hat{\sigma}_{ab\to\gamma,\,N}(\alpha_s)$ for the direct processes. The cross sections for the partonic channels $ab = q\bar{q}', \bar{q}q', qq, qq', \bar{q}\bar{q}, \bar{q}\bar{q}'$ (q and q' denote quarks of different flavours) vanish at LO and are hence suppressed by a factor of α_s with respect to $\hat{\sigma}_{q\bar{q}\to\gamma,\,N}(\alpha_s)$, $\hat{\sigma}_{qg\to\gamma,\,N}(\alpha_s)$, $\hat{\sigma}_{\bar{q}g\to\gamma,\,N}(\alpha_s)$. Moreover, in the large-N limit this relative suppression is furtherly enhanced by a factor of $\mathcal{O}(1/N)$ because the photon has to be accompanied by (at least) two final-state fermions that are not produced by the decay of an off-shell gluon. Therefore, we make no attempt to resum soft-gluon corrections to these partonic channels. The partonic cross section $\hat{\sigma}_{gg\to\gamma,\,N}(\alpha_s)$ has a different large-N behaviour. It begins to contribute at NLO via the partonic process $g+g\to\gamma+q+\bar{q}$, which again leads to a suppression effect of $\mathcal{O}(1/N)$ with respect to the LO subprocesses. However, owing to the photon-gluon coupling through a fermion box, the partonic subprocess $g+g\to\gamma+g$ is also permitted. This subprocess is logarithmically-enhanced by multiple soft-gluon radiation in the final state, but it starts to contribute only at NNLO in perturbation theory. It follows that the partonic channel $ab = gg$ is suppressed by a factor of α_s^2 with respect to the LO partonic channels $ab = q\bar{q}, qg, \bar{q}g$ and it enters the resummed cross section only at NNLL accuracy.

In conclusion, since we are interested in explicitly perform soft-gluon resummation up to NLL order, we can limit ourselves to considering the partonic cross sections $\hat{\sigma}_{q\bar{q}\to\gamma}$, $\hat{\sigma}_{qg\to\gamma}$, $\hat{\sigma}_{\bar{q}g\to\gamma}$.

As discussed in sect. 2, the soft-gluon corrections to the partonic channels $ab = q\bar{q}, qg, \bar{q}g$ are not affected by colour correlations. Thus, in the resummed expressions $\hat{\sigma}^{(\mathrm{res})}_{ab\to\gamma,\,N}$ for the partonic cross sections, the logarithmically-enhanced threshold contributions can be factorized with respect to the corresponding LO cross sections $\hat{\sigma}^{(0)}_{ab\to d\gamma,\,N}$ in eqs. (3.18), (3.19). The *all-order* resummation formulae are

$$\begin{aligned}\hat{\sigma}^{(\mathrm{res})}_{q\bar{q}\to\gamma,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2) = {}& \alpha\,\alpha_s(\mu^2)\,\hat{\sigma}^{(0)}_{q\bar{q}\to g\gamma,\,N}\,C_{q\bar{q}\to\gamma}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2)\,\times \\ & \times\Delta^{q\bar{q}\to g\gamma}_{N+1}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2)\,, \end{aligned} \tag{4.1}$$

$$\hat{\sigma}^{(\mathrm{res})}_{qg\to\gamma,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2) = \alpha\,\alpha_s(\mu^2)\,\hat{\sigma}^{(0)}_{qg\to q\gamma,\,N}\, C_{qg\to\gamma}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) \times \\ \times \Delta^{qg\to q\gamma}_{N+1}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2)\,, \tag{4.2}$$

$$\hat{\sigma}^{(\mathrm{res})}_{\bar{q}g\to\gamma,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2) = \hat{\sigma}^{(\mathrm{res})}_{qg\to\gamma,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2)\,, \tag{4.3}$$

where

$$Q^2 = 2E_T^2\,. \tag{4.4}$$

Note that the right-hand side of eqs. (4.1), (4.2) does not depend on the factorization scale μ_f of the photon fragmentation functions. Thus, the resummed partonic cross sections $\hat{\sigma}^{(\mathrm{res})}_{ab\to\gamma,\,N}$ turn out to be independent of μ_f. This is in agreement with the subdominance of the fragmentation contributions near threshold, as discussed above.

The functions $C_{ab\to\gamma}(\alpha_s)$ in eqs. (4.1), (4.2) do not depend on N and, thus, contain all the contributions that are constant in the large-N limit. These functions are computable as power series expansions in α_s

$$C_{ab\to\gamma}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) = 1 + \sum_{n=1}^{+\infty} \left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n C^{(n)}_{ab\to\gamma}(Q^2/\mu^2; Q^2/\mu_F^2)\,. \tag{4.5}$$

The physical origin and the structure of the constant factors $C_{ab\to\gamma}(\alpha_s)$ is discussed in sect. 4.3.

The $\ln N$-dependence of the resummed cross sections is entirely embodied by the radiative factors $\Delta_N^{ab\to d\gamma}$ on the right-hand side of eqs. (4.1), (4.2). Note, the mismatch between the moment index of the radiative factor and that of $\hat{\sigma}^{(0)}_{ab\to d\gamma,\,N}$: the former depends on $N+1$, like the parton densities in eq. (3.14). The explicit expressions of the radiative factors are given in the following subsection.

4.2. The radiative factors

The soft-gluon factors $\Delta_N^{ab\to d\gamma}$ depend on the flavour of the QCD partons a, b, d involved in the LO hard-scattering subprocess $a + b \to d + \gamma$. According to the discussion of sect. 2, the resummed expressions for $\Delta_N^{ab\to d\gamma}$ have an exponential form. To explain the exponentiation structure and to facilitate the comparison with other hadroproduction processes, we use a notation similar to that in ref. [32] and we write the prompt-photon radiative factors as follows

$$\Delta_N^{ab\to d\gamma}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) = J_N^d(\alpha_s(\mu^2), Q^2/\mu^2)\,\Delta_N^{(\mathrm{int})\,ab\to d\gamma}(\alpha_s(\mu^2), Q^2/\mu^2) \times \\ \times \Delta_N^a(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2)\,\Delta_N^b(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2)\,. \tag{4.6}$$

The resummed formulae to NLL accuracy for the various contributions on the right-hand side of this equation are presented below.

Each term $\Delta_N^a(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2)$ depends on the flavour a of a single parton, on the factorization scheme of the parton density $f_{a/H,\,N}(\mu_F^2)$ and on the factorization scale μ_F. In the $\overline{\mathrm{MS}}$ scheme, we have

$$\Delta_N^a(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) = \\ = \exp\left\{ \int_0^1 dz \, \frac{z^{N-1}-1}{1-z} \int_{\mu_F^2}^{(1-z)^2 Q^2} \frac{dq^2}{q^2} \, A_a(\alpha_s(q^2)) + \mathcal{O}(\alpha_s(\alpha_s \ln N)^k) \right\} , \tag{4.7}$$

where $A_a(\alpha_s)$ are perturbative functions

$$A_a(\alpha_s) = \frac{\alpha_s}{\pi} A_a^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 A_a^{(2)} + \mathcal{O}(\alpha_s^3) \, . \tag{4.8}$$

The lower-order terms $A_a^{(1)}$ and $A_a^{(2)}$ are

$$A_a^{(1)} = C_a \, , \qquad A_a^{(2)} = \frac{1}{2} \, C_a K \, , \tag{4.9}$$

where $C_a = C_F$ if $a = q, \bar{q}$ and $C_a = C_A$ if $a = g$, while the coefficient K is the same both for quarks [39] and for gluons [40] and it is given by[4]

$$K = C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9} T_R N_f \, . \tag{4.10}$$

The term $J_N^d(\alpha_s(\mu^2), Q^2/\mu^2)$ depends on the parton flavour d and is independent both of the factorization scale and of the factorization scheme:

$$J_N^d(\alpha_s(\mu^2), Q^2/\mu^2) = \exp\left\{ \int_0^1 dz \, \frac{z^{N-1}-1}{1-z} \Big[\int_{(1-z)^2 Q^2}^{(1-z) Q^2} \frac{dq^2}{q^2} \, A_d(\alpha_s(q^2)) + \\ + \frac{1}{2} \, B_d(\alpha_s((1-z)Q^2)) \Big] + \mathcal{O}(\alpha_s(\alpha_s \ln N)^k) \right\} . \tag{4.11}$$

The functions $A_d(\alpha_s)$ are given in eq. (4.8) and the functions $B_d(\alpha_s)$ have analogous perturbative expansions:

$$B_d(\alpha_s) = \frac{\alpha_s}{\pi} B_d^{(1)} + \mathcal{O}(\alpha_s^2) \tag{4.12}$$

with [39, 40]

$$B_{d=q,\bar{q}}^{(1)} = -\frac{3}{2} C_F \, , \qquad B_{d=g}^{(1)} = -\frac{1}{6} \, (11 C_A - 4 T_R N_f) \, . \tag{4.13}$$

Likewise J_N^d, the remaining contribution $\Delta_N^{(\mathrm{int})}$ in eq. (4.6) is independent of the factorization scale and scheme. Nonetheless, it depends on the flavours of all the QCD partons entering the LO scattering subprocess:

$$\Delta_N^{(\mathrm{int}) \, ab \to d\gamma}(\alpha_s(\mu^2), Q^2/\mu^2) = \exp\left\{ \int_0^1 dz \, \frac{z^{N-1}-1}{1-z} \, D_{ab \to d\gamma}(\alpha_s((1-z)^2 Q^2)) + \\ + \mathcal{O}(\alpha_s(\alpha_s \ln N)^k) \right\} . \tag{4.14}$$

[4] In $SU(N_c)$ QCD, the colour factors are $C_F = (N_c^2 - 1)/(2N_c), C_A = N_c$ and $T_R = 1/2$.

The function $D_{ab\to d\gamma}(\alpha_s)$ has the following perturbative expansion

$$D_{ab\to d\gamma}(\alpha_s) = \frac{\alpha_s}{\pi} D^{(1)}_{ab\to d\gamma} + \mathcal{O}(\alpha_s^2)\,, \tag{4.15}$$

with

$$D^{(1)}_{ab\to d\gamma} = (C_a + C_b - C_d)\ln 2\,. \tag{4.16}$$

The factorized structure in eq. (4.6) has a direct physical interpretation. The factors Δ^a_N and Δ^b_N take into account soft-gluon radiation emitted collinearly to the *initial*-state partons. Consistently, these are the sole factors that depend on the factorization scale μ_F of the parton densities of the colliding hadrons. The factor J^d_N is due to collinear (either soft or hard) radiation in the *final*-state jet that is produced by the fragmentation of the parton d recoiling against the triggered photon. The factor $\Delta^{(\mathrm{int})}_N$ contains the contribution of soft-gluon emission at large angle with respect to the direction of the hard partons entering the LO scattering subprocess. This factor thus embodies the soft-gluon *interference* effects anticipated in sect. 2.

According to this interpretation, the perturbative functions in eqs. (4.8), (4.12), (4.15) measure the intensity of the coupling of *i*) *soft-collinear* gluons (function $A_a(\alpha_s)$), *ii*) *hard-collinear* partons (function $B_a(\alpha_s)$) and *iii*) *large-angle soft* gluons (function $D_{ab\to d\gamma}(\alpha_s)$). Note that, due to their collinear nature, the functions $A_a(\alpha_s)$ and $B_a(\alpha_s)$ depend on the colour and flavour of the sole parton a. On the contrary, $D_{ab\to d\gamma}(\alpha_s)$ depends on the colour charges of all the QCD partons.

The physical origin of the several contributions on the right-hand side of eq. (4.6) is furtherly discussed in sect. 5, where we compare the promp-photon radiative factors with the analogous resummed factors that control the threshold behaviour of other hadroproduction processes. In the rest of this section we limit ourselves to comment on few additional features of the resummed contributions to the prompt-photon cross section.

The various factors in eq. (4.6) contribute to the resummed prompt-photon cross section at different level of logarithmic accuracy. If we simply consider the double-logarithmic (DL) approximation, which consists in resumming only the terms $\alpha_s^n \ln^{2n} N$, we can neglect the interference factor $\Delta^{(\mathrm{int})}_N$ and the $B(\alpha_s)$ function in eq. (4.11) and we can expand the exponent in Δ^a_N and J^d_N to its first order in α_s:

$$\Delta^a_N(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) \simeq \exp\left\{ + 2C_a \frac{\alpha_s}{2\pi}\ln^2 N\right\}\,, \tag{4.17}$$

$$J^d_N(\alpha_s(\mu^2), Q^2/\mu^2) \simeq \exp\left\{ - C_d \frac{\alpha_s}{2\pi}\ln^2 N\right\}\,. \tag{4.18}$$

The complete set of LL terms is obtained by neglecting the functions $B(\alpha_s), D(\alpha_s)$ in eqs. (4.11), (4.14), by truncating $A_a(\alpha_s)$ to their first order and using the LO running of the coupling $\alpha_s(q^2)$. At the NLL order, also the contribution of the coefficients $A^{(2)}_a, B^{(1)}_a$ and $D^{(1)}_{ab\to d\gamma}$ has to be included.

Note that different scales, e.g. $q^2, (1-z)^2Q^2, (1-z)Q^2$, appear on the right-hand sides of eqs. (4.7), (4.11), (4.14). In particular, the scales in the q^2-integration limits of eq. (4.7) are different from those of eq. (4.11), and the B function in eq. (4.11) depends on $\alpha_s((1-z)Q^2)$ while the D function in the interference contribution (4.14) depends on $\alpha_s((1-z)^2Q^2)$. These scales follows from the hard-scattering kinematics, which affect in a different way initial- or final-state emission and collinear or soft radiation.

Note, also, that the renormalization scale μ does not explicitly enter the right-hand side of eqs. (4.7), (4.11), (4.14). This is because radiative factors are renormalization-group-invariant quantities when evaluated to all order in perturbation theory. Only when the all-order expressions are truncated to a certain degree of logarithmic accuracy, the renormalization-scale dependence explicitly appears as a higher-order effect.

Since we know the radiative factors only to NLL order, we use the eqs. (4.7), (4.11), (4.14) by replacing $\alpha_s(k^2)$ (with $k^2 = q^2, (1-z)^2Q^2, (1-z)Q^2$) with its NLO expansion in terms of $\alpha_s(\mu^2)$ and k^2 (cf. Appendix A), and we explicitly carry out the z and q^2 integrals by neglecting terms beyond NLL accuracy. We thus write the prompt-photons radiative factors as follows:

$$\Delta_N^{ab\to d\gamma}\left(\alpha_s(\mu^2), \frac{Q^2}{\mu^2}; \frac{Q^2}{\mu_F^2}\right) = \exp\Big\{ \ln N\, g_{ab}^{(1)}(b_0\alpha_s(\mu^2)\ln N) \\ + g_{ab}^{(2)}(b_0\alpha_s(\mu^2)\ln N, Q^2/\mu^2; Q^2/\mu_F^2) + \mathcal{O}(\alpha_s(\alpha_s \ln N)^k)\Big\} . \qquad (4.19)$$

The functions $g^{(1)}$ and $g^{(2)}$ resum the LL and NLL terms, respectively. These functions are different for the $q\bar{q}$ and qg partonic channels of eqs. (4.1) and (4.2), and are explicitly computed in Appendix A. We find

$$\begin{aligned} g_{q\bar{q}}^{(1)}(\lambda) &= (2C_F - C_A)\, h^{(1)}(\lambda) + C_A\, h^{(1)}(\lambda/2)\,, \\ g_{qg}^{(1)}(\lambda) &= C_A\, h^{(1)}(\lambda) + C_F\, h^{(1)}(\lambda/2)\,, \end{aligned} \qquad (4.20)$$

and

$$\begin{aligned} g_{q\bar{q}}^{(2)}\left(\lambda, \frac{Q^2}{\mu^2}; \frac{Q^2}{\mu_F^2}\right) &= (2C_F - C_A)\, h^{(2)}(\lambda) + 2\, C_A\, h^{(2)}(\lambda/2) + \\ &+ \frac{2C_F - C_A}{2\pi b_0} \ln 2 \ln(1-2\lambda) + \frac{C_A\gamma_E - \pi b_0}{\pi b_0} \ln(1-\lambda) - \frac{2C_F}{\pi b_0}\lambda \ln\frac{Q^2}{\mu_F^2} + \\ &+ \left\{ \frac{C_F}{\pi b_0}\Big[2\lambda + \ln(1-2\lambda)\Big] + \frac{C_A}{2\pi b_0}\Big[2\ln(1-\lambda) - \ln(1-2\lambda)\Big]\right\} \ln\frac{Q^2}{\mu^2}\,, \end{aligned} \qquad (4.21)$$

$$\begin{aligned} g_{qg}^{(2)}\left(\lambda, \frac{Q^2}{\mu^2}; \frac{Q^2}{\mu_F^2}\right) &= C_A\, h^{(2)}(\lambda) + 2\, C_F\, h^{(2)}(\lambda/2) + \\ &+ \frac{C_A}{2\pi b_0} \ln 2 \ln(1-2\lambda) + \frac{4C_F\,\gamma_E - 3C_F}{4\pi b_0} \ln(1-\lambda) - \frac{C_F + C_A}{\pi b_0}\lambda \ln\frac{Q^2}{\mu_F^2} + \\ &+ \left\{ \frac{C_F + C_A}{2\pi b_0}\Big[2\lambda + \ln(1-2\lambda)\Big] + \frac{C_F}{2\pi b_0}\Big[2\ln(1-\lambda) - \ln(1-2\lambda)\Big]\right\} \ln\frac{Q^2}{\mu^2}\,, \end{aligned} \qquad (4.22)$$

where $\gamma_E = 0.5772\ldots$ is the Euler number and b_0, b_1 are the first two coefficients of the QCD β-function

$$b_0 = \frac{11C_A - 4T_R N_f}{12\pi}\,, \qquad b_1 = \frac{17C_A^2 - 10C_A T_R N_f - 6C_F T_R N_f}{24\pi^2}\,. \qquad (4.23)$$

The auxiliary functions $h^{(1)}$ and $h^{(2)}$ in eqs. (4.20) and (4.21), (4.22) are

$$h^{(1)}(\lambda) = \frac{1}{2\pi b_0 \lambda}\Big[2\lambda + (1-2\lambda)\ln(1-2\lambda)\Big] \,, \tag{4.24}$$

$$\begin{aligned} h^{(2)}(\lambda) = & \frac{b_1}{2\pi b_0^3}\Big[2\lambda + \ln(1-2\lambda) + \frac{1}{2}\ln^2(1-2\lambda)\Big] - \\ & - \frac{\gamma_E}{\pi b_0}\ln(1-2\lambda) - \frac{K}{4\pi^2 b_0^2}\Big[2\lambda + \ln(1-2\lambda)\Big] \,, \end{aligned} \tag{4.25}$$

where K is the coefficient in eq. (4.10).

The results in eqs. (4.19)–(4.22) provide us with a theoretical description of soft-gluon resummation in prompt-photon hadroproduction at the same level of accuracy as for other hadroproduction processes, such as the production of Drell-Yan pairs [31, 32] or heavy quarks [36, 28]. These results can be used for detailed quantitative studies along the lines of Refs. [41, 28]. In this paper we do not present numerical analyses and we limit ourselves to discuss the expected sign and size of the resummation effects.

In the near-threshold region, radiation in the final state is kinematically inhibited. On physical basis, one thus expects that resummation of the ensuing logarithmically-enhanced corrections produces suppression of the cross section. This argument applies to *hadronic* cross sections, but it is not necessarely valid for *partonic* cross sections. The partonic cross section is what is left after factorization of long-distance physics into the parton distributions. Since all-order resummation is in part automatically implemented in the definition of the parton densities, the remaining resummation effects can either enhance or deplete the partonic cross section.

Among the various terms on the right-hand side of eq. (4.6), some factors are smaller and some others are larger than unity. The exponent of the initial-state contribution Δ_N^a in eq.(4.7) is positive definite and, hence, $\Delta_N^a \gg 1$ when $N \to \infty$. The presence of this 'anti-Sudakov' form factor is typical of partonic cross sections that are evaluated by factorizing parton densities defined in the $\overline{\mathrm{MS}}$ factorization scheme. In the case of the final-state contribution J_N^d, no additional factorization has been performed. Therefore, when $N \to \infty$ the exponent in eq. (4.11) is negative and $J_N^d \ll 1$ is a 'true' Sudakov form factor, as naively expected. The sign of the exponent in eq. (4.14) is not definite ($D^{(1)}_{q\bar{q}\to g\gamma} < 0, D^{(1)}_{qg\to q\gamma} = D^{(1)}_{\bar{q}g\to \bar{q}\gamma} > 0$) as expected for an interference term. However, the contribution of $\Delta_N^{(\mathrm{int})\, ab\to d\gamma}$ is subleading with respect to those of Δ_N^a and J_N^d.

From the overall inspection of the effect of the radiative-factor contributions to eq. (4.6), we infer that, in the case of prompt-photon production, the resummed partonic cross sections $\hat{\sigma}^{(\mathrm{res})}_{q\bar{q}\to\gamma,\, N}$ and $\hat{\sigma}^{(\mathrm{res})}_{qg\to\gamma,\, N}$ in eqs. (4.1) and (4.2) are both enhanced with respect to their LO approximations $\hat{\sigma}^{(0)}_{q\bar{q}\to g\gamma,\, N}$, $\hat{\sigma}^{(0)}_{qg\to q\gamma,\, N}$. Moreover, the enhancement in the qg partonic channel is larger than that in the $q\bar{q}$ channel.

This conclusion can also be argued by a simplified treatment within the DL approximation. Inserting eqs. (4.17), (4.18) into eq. (4.6), we obtain

$$\hat{\sigma}^{(\rm res)}_{qg\to\gamma,\,N} \simeq \hat{\sigma}^{(0)}_{qg\to q\gamma,\,N}\;\exp\left\{\left[2C_F+2C_A-C_F\right]\frac{\alpha_s}{2\pi}\ln^2 N\right\} \tag{4.26}$$

$$= \hat{\sigma}^{(0)}_{qg\to q\gamma,\,N}\;\exp\left\{(C_F+2C_A)\,\frac{\alpha_s}{2\pi}\ln^2 N\right\} > \hat{\sigma}^{(0)}_{qg\to q\gamma,\,N}\,, \tag{4.27}$$

$$\hat{\sigma}^{(\rm res)}_{q\bar{q}\to\gamma,\,N} \simeq \hat{\sigma}^{(0)}_{q\bar{q}\to q\gamma,\,N}\;\exp\left\{\left[2C_F+2C_F-C_A\right]\frac{\alpha_s}{2\pi}\ln^2 N\right\} \tag{4.28}$$

$$= \hat{\sigma}^{(0)}_{q\bar{q}\to q\gamma,\,N}\;\exp\left\{(4C_F-C_A)\,\frac{\alpha_s}{2\pi}\ln^2 N\right\} > \hat{\sigma}^{(0)}_{q\bar{q}\to q\gamma,\,N}\,, \tag{4.29}$$

$$\frac{\hat{\sigma}^{(\rm res)}_{qg\to\gamma,\,N}}{\hat{\sigma}^{(\rm res)}_{q\bar{q}\to\gamma,\,N}} \simeq \frac{\hat{\sigma}^{(0)}_{qg\to q\gamma,\,N}}{\hat{\sigma}^{(0)}_{q\bar{q}\to q\gamma,\,N}}\;\exp\left\{3(C_A-C_F)\frac{\alpha_s}{2\pi}\ln^2 N\right\} > \frac{\hat{\sigma}^{(0)}_{qg\to q\gamma,\,N}}{\hat{\sigma}^{(0)}_{q\bar{q}\to q\gamma,\,N}}\,. \tag{4.30}$$

The first, second and third terms in the square bracket on the right-hand side of eqs. (4.26), (4.28) are respectively due to the initial-state factors Δ^a_N, Δ^b_N and to the final-state factor J^d_N. Note that, for a definite parton a, the initial-state enhancement Δ^a_N is larger than the final-state suppression J^a_N (see the difference by a factor of two in the exponent of eqs. (4.17), (4.18)). In the qg channel the final-state contribution $\ln J^q_N$ is thus overcompensated by $\ln\Delta^q_N$ and this leads to the enhancement in eq. (4.27). In the $q\bar{q}$ channel, instead, it is the total initial-state contribution $(\ln\Delta^q_N + \ln\Delta^{\bar{q}}_N)$ that, owing to the colour-charge relation $C_F \sim C_A/2$, overcompensates $\ln J^g_N$. Finally, the enhancement in eq. (4.30) is simply due the fact that the gluon colour charge C_A is larger that the quark charge C_F and, thus, $\Delta^g_N > \Delta^q_N$ and $J^q_N > J^g_N$.

Note that this conclusion directly applies only to the asymptotic limit $N\to\infty$ or $E_T \to \sqrt{S}/2$. In the case of kinematic configurations of experimental interest, subleading effects and their dependence on the x-shape of the parton distributions and on the renormalization and factorization scale have to be carefully estimated.

4.3. The constant factors

Expanding the resummed expressions in eqs.(4.19)–(4.22) to the first order in α_s and using eqs. (4.1), (4.2), we obtain

$$\begin{aligned}
\hat{\sigma}^{(\rm res)}_{q\bar{q}\to\gamma,\,N}(\alpha_s(\mu^2);E_T^2,\mu^2,\mu_F^2,\mu_f^2) &= \alpha\,\hat{\sigma}^{(0)}_{q\bar{q}\to g\gamma,\,N}\,\alpha_s(\mu^2)\left\{1+\frac{\alpha_s(\mu^2)}{\pi}\left[\left(2C_F-\frac{1}{2}C_A\right)\ln^2 N\right.\right.\\
&+\left(\gamma_E(4C_F-C_A)-(2C_F-C_A)\ln 2+\pi b_0-2C_F\ln\frac{2E_T^2}{\mu_F^2}\right)\ln N\\
&\left.\left.+C^{(1)}_{q\bar{q}\to\gamma}(2E_T^2/\mu^2;2E_T^2/\mu_F^2)\right]+\mathcal{O}(\alpha_s^2)\right\},
\end{aligned} \tag{4.31}$$

$$\begin{aligned}
\hat{\sigma}^{(\rm res)}_{qg\to\gamma,\,N}(\alpha_s(\mu^2);E_T^2,\mu^2,\mu_F^2,\mu_f^2) &= \alpha\,\hat{\sigma}^{(0)}_{qg\to q\gamma,\,N}\,\alpha_s(\mu^2)\left\{1+\frac{\alpha_s(\mu^2)}{\pi}\left[\left(\frac{1}{2}C_F+C_A\right)\ln^2 N+\right.\right.\\
&+\left(\gamma_E(C_F+2C_A)-C_A\ln 2+\frac{3}{4}C_F-(C_F+C_A)\ln\frac{2E_T^2}{\mu_F^2}\right)\ln N+\\
&\left.\left.+C^{(1)}_{qg\to\gamma}(2E_T^2/\mu^2;2E_T^2/\mu_F^2)\right]+\mathcal{O}(\alpha_s^2)\right\}.
\end{aligned} \tag{4.32}$$

One can easily check that the logarithmic terms in these perturbative expansions agree with those that can be derived from the complete NLO analytic results of Refs. [1, 42, 3]. From this comparison we can also extract the first-order constant coefficients $C^{(1)}_{q\bar{q}\to\gamma}$ and $C^{(1)}_{qg\to\gamma}$. We find

$$C^{(1)}_{q\bar{q}\to\gamma}(Q^2/\mu^2;Q^2/\mu_F^2) = \gamma_E^2\Big(2C_F - \frac{1}{2}C_A\Big) + \gamma_E\Big[\pi b_0 - (2C_F - C_A)\ln 2\Big] - \\ - \frac{1}{2}(2C_F - C_A)\ln 2 + \frac{1}{2}K - K_q + \frac{\pi^2}{3}\Big(2C_F - \frac{1}{2}C_A\Big) + \frac{5}{4}(2C_F - C_A)\ln^2 2 - \\ - \Big(2\gamma_E C_F - \frac{3}{2}C_F\Big)\ln\frac{Q^2}{\mu_F^2} - \pi b_0 \ln\frac{Q^2}{\mu^2}\,, \tag{4.33}$$

$$C^{(1)}_{qg\to\gamma}(Q^2/\mu^2;Q^2/\mu_F^2) = \gamma_E^2\Big(\frac{1}{2}C_F + C_A\Big) + \gamma_E\Big[\frac{3}{4}C_F - C_A\ln 2\Big] - \\ - \frac{1}{10}(C_F - 2C_A)\ln 2 - \frac{1}{2}K_q + \frac{\pi^2}{60}\Big(2C_F + 19C_A\Big) + \frac{1}{2}C_F\ln^2 2 - \\ - \Big(\gamma_E(C_F + C_A) - \frac{3}{4}C_F - \pi b_0\Big)\ln\frac{Q^2}{\mu_F^2} - \pi b_0\ln\frac{Q^2}{\mu^2}\,, \tag{4.34}$$

where

$$K_q = \left(\frac{7}{2} - \frac{\pi^2}{6}\right) C_F\,, \tag{4.35}$$

and the coefficient K is given in eq. (4.10).

The first-order coefficient $C^{(1)}_{ab\to\gamma}$ and, indeed, all the perturbative coefficients of the constant (N-independent) function $C_{ab\to\gamma}(\alpha_s)$ in eq. (4.5) are produced by hard *virtual* contributions and by subdominant (non-logarithmic) soft corrections to the LO hard-scattering subprocesses. In both cases the structure of the external hard partons is the same as at LO. This justifies the all-order factorization of $C_{ab\to\gamma}(\alpha_s)$ with respect to $\hat{\sigma}^{(0)}_{ab\to d\gamma,\,N}$ and to the radiative factor in the resummed partonic cross sections (4.1, 4.2).

The inclusion of the N-independent function $C_{ab\to\gamma}(\alpha_s)$ in the resummed formulae does not affect the shape of the cross section near threshold, but improves the soft-gluon resummation by fixing the overall (perturbative) normalization of the logarithmic radiative factor.

We can explicitly show [43, 28] the theoretical improvement that is obtained by combining the NLL radiative factor with the first-order coefficient $C^{(1)}$. Expanding the resummation formulae (4.1), (4.2) in towers of logarithmic contributions as in eq. (2.1), we have

$$\hat{\sigma}^{(\mathrm{res})}_N(\alpha_s;E_T^2,\mu^2,\mu_F^2) = \alpha\,\alpha_s\,\hat{\sigma}^{(0)}_N\Big\{1 + \sum_{n=1}^{\infty}\alpha_s^n\Big[c_{n,2n}\,\ln^{2n}N + c_{n,2n-1}(E_T^2/\mu_F^2)\,\ln^{2n-1}N + \\ + c_{n,2n-2}(E_T^2/\mu_F^2,E_T^2/\mu^2)\,\ln^{2n-2}N + \mathcal{O}(\ln^{2n-3}N)\Big]\Big\}\,, \tag{4.36}$$

where $\alpha_s = \alpha_s(\mu^2)$. The dominant and next-to-dominant coefficients $c_{n,2n}$ and $c_{n,2n-1}$ are controlled by evaluating the radiative factor to NLL accuracy. When the NLL radiative factor is supplemented with the coefficient $C^{(1)}$, we can correctly control also the coefficients $c_{n,2n-2}$. In particular, we can predict (see Appendix B) the large-N behaviour of the NNLO cross sections $\hat{\sigma}^{(2)}_{ab\to\gamma}$ in eq. (3.6) up to $\mathcal{O}(\ln N)$.

Note also that coefficients $c_{n,2n}$ are scale independent and the coefficients $c_{n,2n-1}$ depend on the sole factorization scale μ_F. In the tower expansion (4.36), the first terms that explicitly depend on the renormalization scale μ (and on μ_F, as well) are those controlled by $c_{n,2n-2}$. Their dependence on μ is obtained by combining that of $C^{(1)}(E_T^2/\mu_F^2, E_T^2/\mu^2)$ with that of the radiative factor at NLL order. The inclusion of the first-order constant coefficient $C^{(1)}$ thus (theoretically) stabilizes the resummed partonic cross section with respect to variations of the renormalization scale.

5. Comparison with other processes: soft-gluon interferences and QCD coherence

Further insight on the underlying physics mechanism that leads to the resummed expressions (4.1), (4.2) can be obtained by comparing prompt-photon production with other hard-scattering processes.

In the hadroproduction of a DY lepton pair (Fig. 1a) of high mass Q^2, the vicinity to the threshold region is measured by the inelasticity variable $\tau = Q^2/S$, where $\sqrt{S}$ is the centre-of-mass energy. The Born-level partonic process that controls the cross section is $q\bar{q}$ annihilation. In N-moment space, where the N-moments are defined with respect to τ, the Sudakov corrections to the $q\bar{q}$-annihilation cross section are taken into account by a resummation formula analogous to eqs. (4.1), (4.2). Up to NLL accuracy, the corresponding radiative factor $\Delta_{DY,\,N}(Q^2)$ has the following explicit expression [31, 32]

$$\Delta_{DY,\,N}(Q^2) = \Delta^q_N(Q^2)\,\Delta^{\bar{q}}_N(Q^2)\,, \tag{5.1}$$

where $\Delta^q_N(Q^2)$ and $\Delta^{\bar{q}}_N(Q^2)$ are the single-parton contributions[5] given in eq. (4.7). Each term Δ^a_N embodies multiple initial-state radiation of soft gluons, i.e. gluons that carry a small fraction $1-z \sim 1/N \sim (1-\tau)$ of the energy of the initial-state parton a. The factorized structure on the right-hand side of eq. (5.1) implies that soft-gluon interferences between the two hard partons cancel to this logarithmic accuracy [44].

Note that this cancellation does not depend on the type of annihilating partons. In fact, when the DY pair is replaced by a colourless system, say, a Higgs boson, produced by gluon-gluon fusion, the resummed partonic cross section is controlled by a NLL radiative factor [40, 45]

$$\Delta_{Higgs,\,N}(Q^2) = \Delta^g_N(Q^2)\,\Delta^g_N(Q^2)\,, \tag{5.2}$$

which is again factorized in single-parton contributions.

[5]To simplify the notation, we drop the explicit dependence on α_s and on the renormalization and factorization scale. Therefore, we use $\Delta^a_N(Q^2) \equiv \Delta^q_N(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2)$ and $J^a_N(Q^2) \equiv J^a_N(\alpha_s(\mu^2), Q^2/\mu^2)$ throughout this Section.

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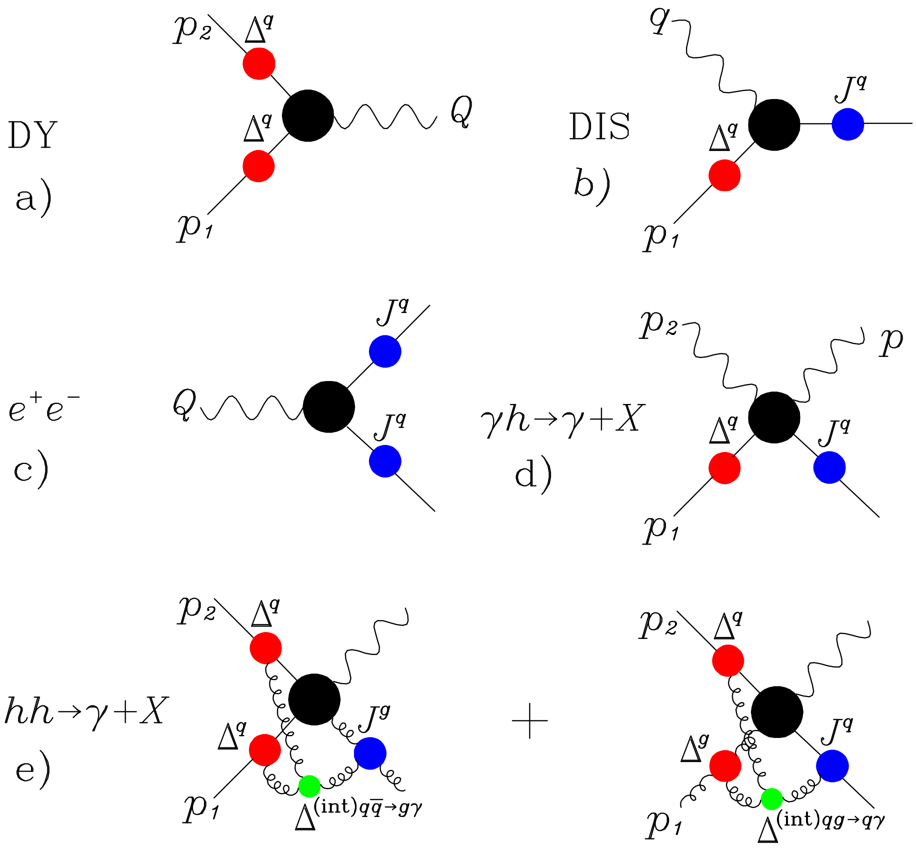

Figure 1: Schematic representation of the structure of the soft-radiation factors in: (a) DY production, (b) DIS, (c) e^+e^- annihilation, (d) prompt-photon photoproduciton and (e) prompt-photon hadroproduction.

The presence of non-interfering Sudakov factors is typical of other processes dominated by hard scattering of two QCD partons, such as lepton-hadron DIS, e^+e^- annihilation in two jets and prompt-photon photoproduction.

In the case of inclusive DIS (Fig. 1b), the hard-scattering scale $Q^2 = -q^2$ is given by the square of the space-like transferred momentum q and the relevant inelasticity variable is the Bjorken variable $x_{Bj} = Q^2/2P_1 \cdot q$. The Born-level partonic process is lepton-quark scattering and, when the threshold region $x_{Bj} \to 1$ is approched, the corresponding radiative factor $\Delta_{DIS,\,N}(Q^2)$ in N-moment space is [44, 46]

$$\Delta_{DIS,\,N}(Q^2) = \Delta_N^q(Q^2)\, J_N^q(Q^2). \tag{5.3}$$

The Sudakov factor $\Delta_N^q(Q^2)$ is exactly the same as in the DY process. It embodies soft-gluon radiation from the initial-state quark. Unlike in the DY process, however, in DIS the scattered initial-state quark fragments in the final state. Then the factor $J_N^q(Q^2)$ takes into account the fragmentation of the final-state quark into a jet of collinear and/or soft partons with a small invariant mass $k^2 \sim Q^2/N \sim (1 - x_{Bj})Q^2$. The NLL expression of the jet mass distribution $J_N^a(Q^2)$ is given in eq. (4.11).

Hadronic final states with two back-to-back jets produced in e^+e^- annihilation at the centre-of-mass energy Q (Fig. 1c) are also controlled by the jet mass distribution $J^a_N(Q^2)$ [43]. For instance, in the case of the distribution $(1/\sigma)\, d\sigma/dT$ of the thrust T [47], the Sudakov region is $T \to 1$. In this limit we have $1-T \simeq k_1^2/Q^2 + k_2^2/Q^2$, where k_1^2 and k_2^2 are the hadronic invariant masses in the two emispheres singled out by the plane orthogonal to the thrust axis. Considering the N-moments $\Delta_{T(e^+e^-),\,N}(Q^2)$ of the thrust distribution with respect to T, and taking the large-N limit, one obtains [47]

$$\Delta_{T(e^+e^-),\,N}(Q^2) = J^q_N(Q^2)\, J^{\bar{q}}_N(Q^2)\,. \tag{5.4}$$

The factors J^q and $J^{\bar{q}}$ are the invariant-mass distributions of the two jets that originate from the fragmentation of the $q\bar{q}$-pair produced by the e^+e^--annihilation process at the Born level.

The structure of the radiative factors in eqs. (5.1)–(5.4) easily explains the high-E_T behaviour of the prompt-photon cross section in photoproduction collisions (Fig. 1d). This process, which can be regarded as a simplified case of the hadroproduction process considered throughout the paper, is discussed in Appendix C. In hadron-photon collisions the production of high-E_T prompt photons is dominated at the Born level by the Compton-scattering subprocess $q(x_1P_1) + \gamma(p_2) \to q(p_3) + \gamma(p)$. The all-order resummation of Sudakov effects leads to the radiative factor $\Delta_N^{q\gamma\to q\gamma}$ in eq. (C.8), whose NLL expression is given in eq. (C.12):

$$\Delta_N^{q\gamma\to q\gamma}(Q^2) = \Delta^q_N(Q^2)\, J^q_N(Q^2)\,. \tag{5.5}$$

The factor $\Delta^q_N(Q^2)$ takes into account sof-gluon radiation from the initial-state quark, while $J^q_N(Q^2)$ is the mass distribution of the jet produced by the collinear and/or soft fragmentation of the final-state quark.

Note that high-E_T prompt-photon photoproduction can be regarded as a photon-hadron deep-inelastic scattering, where the space-like momentum transferred by the scattered photon is $q^\mu = p_2^\mu - p^\mu$. Since the high-E_T cross section is dominated by the kinematics configurations in which the prompt photon is produced in the central rapidity region, we have $2p_2\cdot p \simeq 2E_T^2$ and $2P_1\cdot p \simeq P_1\cdot p_2 = S/2$. Thus, the hard scale is $Q^2 = -q^2 = 2p_2\cdot p \simeq 2E_T^2$ and the inelasticity variable analogous to the Bjorken variable is $Q^2/2P_1\cdot q = Q^2/(2P_1\cdot p_2 - 2P_1\cdot p) \simeq 4E_T^2/S = x_T^2$. Recalling that in eq. (5.5) we have $Q^2 = 2E_T^2$ (see eq. (C.10)) and that N is the moment index with respect to x_T^2 (see eq. (C.7)), we can thus straigtforwardly understand the complete analogy between eq. (5.5) and the expression (5.3) for the DIS radiative factor.

The Sudakov corrections to prompt-photon hadroproduction (Fig. 1e) are embodied in eqs. (4.1), (4.2) through the radiative factor $\Delta_N^{ab\to d\gamma}(Q^2)$. On the basis of the factorization of the right-hand side of eqs. (5.1)–(5.5) in terms of initial- and final-state single-parton contributions, one might expect that $\Delta_N^{ab\to d\gamma}$ can be obtained from the photoproduction result in eq. (5.5) by simply including an additional initial-state factor Δ^b_N. The NLL expression (4.6) for $\Delta_N^{ab\to d\gamma}(Q^2)$ shows that this naive expectation is not correct. In fact, we have

$$\Delta_N^{ab\to d\gamma}(Q^2) = \Delta^a_N(Q^2)\, \Delta^b_N(Q^2)\, J^d_N(Q^2)\, \Delta_N^{(\mathrm{int})\, ab\to d\gamma}(Q^2)\,. \tag{5.6}$$

The presence of the NLL contribution $\Delta_N^{(\mathrm{int})\,ab\to d\gamma}$ on the right-hand side of eq. (5.6) implies that the physical picture of the Sudakov radiative factors in terms of *independent* single-parton contributions is not valid, in general. As discussed in sect. 2 and explicitly shown in eqs. (5.1)–(5.5), this picture applies to processes dominated by hard-scattering of two sole partons, but it breaks down at NLL accuracy in the case of multiparton hard-scattering. The breakdown is due to *interferences* and *colour correlations* produced by soft gluons that are radiated at large angle with respect to the directions of the hard-parton momenta [29]. Soft-gluon interferences are present in the hard-scattering of three QCD partons as shown by eq. (5.6), while colour correlations affect hard-scattering of more than three QCD partons [36].

Owing to their large-angle origin, soft-gluon interferences are process dependent. In the case of prompt-photon hadroproduction they are taken into account by the factor $\Delta_N^{(\mathrm{int})\,ab\to d\gamma}$, whose explicit NLL expression is given in eqs. (4.14)–(4.16).

Note that the coefficient $D^{(1)}_{ab\to d\gamma}$ in eq. (4.16) depends linearly on the colour charges of the hard partons, and the colour-charge dependence of $\Delta_N^{(\mathrm{int})\,ab\to d\gamma}$ is thus factorized at NLL accuracy. This suggests that the effect of the interference factor can be absorbed by a proper rescaling of the independent-emission factors Δ^a, Δ^b and J^d. As a matter of fact, neglecting corrections beyond NLL order, one can check that the right-hand side of eq. (5.6) can be rewritten as follows

$$\Delta_N^{ab\to d\gamma}(Q^2) = \Delta^a_{N/2}(Q^2/2)\,\Delta^b_{N/2}(Q^2/2)\,J^d_{N/2}(Q^2/2)\,. \tag{5.7}$$

This equation has to be regarded as a manifestation of the colour-coherence properties of QCD emission [35]. Soft gluons radiated at large angle destructively interfere. Their effect can thus be taken into account by Sudakov factors of independent emission in a restricted (angular) region of the phase space.

6. Conclusion

In this paper we presented the explicit expressions for the resummation of threshold-enhanced logarithms in hadronic prompt-photon production, to next-to-leading accuracy. The simple colour structure of the diagrams contributing to prompt-photon production reflects itself in the simplicity of the resummed formulae. Fragmentation processes, furthermore, do not contribute to the Sudakov resummation at NLL level. In Mellin space, the resummed radiative factor factorizes in the product of three independent contributions for the initial and final coloured partons appearing in the Born process, times a simple factor describing the soft-gluon interferences between initial and final states. General coherence properties of large-angle soft-gluon radiation allow to further simplify the result: the interference contributions can be described, to the same degree of accuracy, by constraining the phase-space for independent emission from the coloured partons. The resulting radiation factor can thus be written as the product of the three independent single-parton contributions, with a properly rescaled dependence on the Mellin-moment variable N.

The formulae are given in terms of Mellin moments, and can be used for phenomenological applications by inverse-Mellin transforming to x_T space. The problems related to this inversion are the same as those encountered in the resummation of the Drell-Yan or heavy-quark production cross-sections, and can therefore be solved with the same techniques [41]. All ingredients are therefore available for a phenomenological study of prompt-photon production including the evaluation of Sudakov effects with NLL accuracy. Such a study is in progress, and will be reported soon. The calculations presented in this work, together with previous work on Drell-Yan and DIS, make it now possible to carry out global fits of parton densities with a uniform NLL accuracy in the large-x region. All of the processes that are used for these global fits, among which prompt-photon production plays a critical role, are now known theoretically at this level of accuracy.

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A. NLL formulae for the radiative factors

The logarithmic expansion of the radiative factors in eqs. (4.7), (4.11), (4.14) can be computed as described in refs. [32, 43]. The running coupling $\alpha_s(k^2)$ with $k^2 = q^2$, $(1-z)^2Q^2, (1-z)Q^2$ has to be expressed in terms of $\alpha_s(\mu^2)$ according to the NLO solution of the renormalization group equation:

$$\alpha_s(k^2) = \frac{\alpha_s(\mu^2)}{1+b_0\alpha_s(\mu^2)\ln(k^2/\mu^2)} \left[1 - \frac{b_1}{b_0}\frac{\alpha_s(\mu^2)\ln(1+b_0\alpha_s(\mu^2)\ln(k^2/\mu^2))}{1+b_0\alpha_s(\mu^2)\ln(k^2/\mu^2)} + \right.$$
$$\left. +\mathcal{O}(\alpha_s^2(\mu^2)(\alpha_s(\mu^2)\ln(k^2/\mu^2))^n) \right], \tag{A.1}$$

where b_0, b_1 are the first two coefficients of the QCD β-function, which are explicitly reported in eq. (4.23). Then the z integration can be performed with NLL accuracy by setting

$$z^{N-1} - 1 \simeq -\Theta(1 - z - e^{-\gamma_E}/N)\,. \tag{A.2}$$

Defining

$$\lambda = b_0\alpha_s(\mu^2)\ln N\,, \tag{A.3}$$

we find

$$\begin{aligned} \ln \Delta_N^a(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) &= \ln N\, h_a^{(1)}(\lambda) + \\ &\quad + h_a^{(2)}(\lambda, Q^2/\mu^2; Q^2/\mu_F^2) + \mathcal{O}\left(\alpha_s(\alpha_s \ln N)^k\right), \quad \text{(A.4)} \\ \ln J_N^a(\alpha_s(\mu^2), Q^2/\mu^2) &= \ln N\, f_a^{(1)}(\lambda) + \\ &\quad + f_a^{(2)}(\lambda, Q^2/\mu^2) + \mathcal{O}\left(\alpha_s(\alpha_s \ln N)^k\right), \quad \text{(A.5)} \end{aligned}$$

$$\ln \Delta_N^{(\mathrm{int})\, ab\to d\gamma}(\alpha_s(\mu^2), Q^2/\mu^2) = \frac{D^{(1)}_{ab\to d\gamma}}{2\pi b_0} \ln(1-2\lambda) + \mathcal{O}\left(\alpha_s(\alpha_s \ln N)^k\right) , \tag{A.6}$$

where the LL and NLL functions $h_a^{(1)}, f_a^{(1)}$ and $h_a^{(2)}, f_a^{(2)}$ are given in terms of the perturbative coefficients $A_a^{(1)}, A_a^{(2)}, B_a^{(1)}$ in eqs. (4.8), (4.12):

$$\begin{aligned} h_a^{(1)}(\lambda) &= + \frac{A_a^{(1)}}{2\pi b_0 \lambda}\Big[2\lambda + (1-2\lambda)\ln(1-2\lambda)\Big] , & \text{(A.7)} \\ h_a^{(2)}(\lambda, Q^2/\mu^2; Q^2/\mu_F^2) &= + \frac{A_a^{(1)} b_1}{2\pi b_0^3}\Big[2\lambda + \ln(1-2\lambda) + \frac{1}{2}\ln^2(1-2\lambda)\Big] - \\ &- \frac{A_a^{(1)}\gamma_E}{\pi b_0}\ln(1-2\lambda) - \\ &- \frac{A_a^{(2)}}{2\pi^2 b_0^2}\Big[2\lambda + \ln(1-2\lambda)\Big] + \\ &+ \frac{A_a^{(1)}}{2\pi b_0}\Big[2\lambda + \ln(1-2\lambda)\Big]\ln\frac{Q^2}{\mu^2} - \frac{A_a^{(1)}}{\pi b_0}\lambda \ln\frac{Q^2}{\mu_F^2} , & \text{(A.8)} \end{aligned}$$

$$\begin{aligned} f_a^{(1)}(\lambda) &= - \frac{A_a^{(1)}}{2\pi b_0 \lambda}\Big[(1-2\lambda)\ln(1-2\lambda) - 2(1-\lambda)\ln(1-\lambda)\Big] , & \text{(A.9)} \\ f_a^{(2)}(\lambda, Q^2/\mu^2) &= - \frac{A_a^{(1)} b_1}{2\pi b_0^3}\Big[\ln(1-2\lambda) - 2\ln(1-\lambda) + \frac{1}{2}\ln^2(1-2\lambda) - \ln^2(1-\lambda)\Big] + \\ &+ \frac{B_a^{(1)}}{2\pi b_0}\ln(1-\lambda) - \frac{A_a^{(1)}\gamma_E}{\pi b_0}\Big[\ln(1-\lambda) - \ln(1-2\lambda)\Big] - \\ &- \frac{A_a^{(2)}}{2\pi^2 b_0^2}\Big[2\ln(1-\lambda) - \ln(1-2\lambda)\Big] + \\ &+ \frac{A_a^{(1)}}{2\pi b_0}\Big[2\ln(1-\lambda) - \ln(1-2\lambda)\Big]\ln\frac{Q^2}{\mu^2} . & \text{(A.10)} \end{aligned}$$

Note that the functions $f_a^{(1)}(\lambda)$ and $f_a^{(2)}(\lambda, Q^2/\mu^2)$ can also be written in terms of $h_a^{(1)}$ and $h_a^{(2)}$ as follows

$$\begin{aligned} f_a^{(1)}(\lambda) &= h_a^{(1)}(\lambda/2) - h_a^{(1)}(\lambda) , & \text{(A.11)} \\ f_a^{(2)}(\lambda, Q^2/\mu^2) &= 2\, h_a^{(2)}(\lambda/2, Q^2/\mu^2; 1) - h_a^{(2)}(\lambda, Q^2/\mu^2; 1) + \\ &+ \frac{B_a^{(1)} + 2A_a^{(1)}\gamma_E}{2\pi b_0}\ln(1-\lambda) . & \text{(A.12)} \end{aligned}$$

Inserting the expressions (A.4), (A.5), (A.6) into eq. (4.6), and using the explicit form of the perturbative coefficients $A_a^{(1)}, A_a^{(2)}, B_a^{(1)}, D^{(1)}_{ab\to d\gamma}$ in eqs. (4.9), (4.13), (4.16), we obtain the results in eqs. (4.19)–(4.22).

B. NNLO partonic cross sections at large N

According to the notation in eq. (4.36), the large-N behaviour of the NLO cross section $\hat{\sigma}^{(1)}_{ab\to\gamma}$ in eq. (3.6) is written as

$$\hat{\sigma}^{(1)}_{ab\to\gamma,\,N}(E_T^2,\mu^2,\mu_F^2,\mu_f^2) = \hat{\sigma}^{(0)}_{ab\to d\gamma,\,N}\Big[c_{1,2}^{(ab)}\,\ln^2 N + c_{1,1}^{(ab)}(E_T^2/\mu_F^2)\ln N + \\ + c_{1,0}^{(ab)}(E_T^2/\mu_F^2, E_T^2/\mu^2) + \mathcal{O}(1/N)\Big]\,, \quad \text{(B.1)}$$

where the various coefficients can be read from eqs. (4.31), (4.32)

$$c_{1,2}^{(q\bar{q})} = \frac{1}{\pi}\left(2C_F - \frac{1}{2}C_A\right), \quad c_{1,2}^{(qg)} = \frac{1}{\pi}\left(\frac{1}{2}C_F + C_A\right), \quad \text{(B.2)}$$

$$c_{1,1}^{(q\bar{q})}(E_T^2/\mu_F^2) = \frac{1}{\pi}\left[\gamma_E(4C_F - C_A) - (2C_F - C_A)\ln 2 + \pi b_0 - 2C_F \ln\frac{2E_T^2}{\mu_F^2}\right],$$
$$c_{1,1}^{(qg)}(E_T^2/\mu_F^2) = \frac{1}{\pi}\left[\gamma_E(C_F + 2C_A) - C_A\ln 2 + \frac{3}{4}C_F - (C_F + C_A)\ln\frac{2E_T^2}{\mu_F^2}\right], \quad \text{(B.3)}$$

$$c_{1,0}^{(q\bar{q})}(E_T^2/\mu_F^2, E_T^2/\mu^2) = \frac{1}{\pi}C^{(1)}_{q\bar{q}\to\gamma}(2E_T^2/\mu^2; 2E_T^2/\mu_F^2)\,,$$
$$c_{1,0}^{(qg)}(E_T^2/\mu_F^2, E_T^2/\mu^2) = \frac{1}{\pi}C^{(1)}_{qg\to\gamma}(2E_T^2/\mu^2; 2E_T^2/\mu_F^2)\,, \quad \text{(B.4)}$$

and $C^{(1)}_{q\bar{q}\to\gamma}, C^{(1)}_{qg\to\gamma}$ are given in eqs. (4.33), (4.34).

Analogously, we can write the NNLO cross section $\hat{\sigma}^{(2)}_{ab\to\gamma}$ as follows:

$$\hat{\sigma}^{(2)}_{ab\to\gamma,\,N}(E_T^2,\mu^2,\mu_F^2,\mu_f^2) = \hat{\sigma}^{(0)}_{ab\to d\gamma,\,N}\Big[c_{2,4}^{(ab)}\,\ln^4 N + c_{2,3}^{(ab)}(E_T^2/\mu_F^2)\ln^3 N + \\ + c_{2,2}^{(ab)}(E_T^2/\mu_F^2, E_T^2/\mu^2)\ln^2 N + \mathcal{O}(\ln N)\Big]\,. \quad \text{(B.5)}$$

The coefficients $c_{2,4}, c_{2,3}, c_{2,2}$ can be calculated by expanding the resummation formulae (4.1), (4.2) to the second order in α_s. We find

$$c_{2,4}^{(ab)} = \frac{1}{2}\left[c_{1,2}^{(ab)}\right]^2, \quad \text{(B.6)}$$

$$c_{2,3}^{(q\bar{q})}(E_T^2/\mu_F^2) = c_{1,2}^{(q\bar{q})}\,c_{1,1}^{(q\bar{q})}(E_T^2/\mu_F^2) + \frac{2}{3\pi}b_0\left(2C_F - \frac{3}{4}C_A\right),$$
$$c_{2,3}^{(qg)}(E_T^2/\mu_F^2) = c_{1,2}^{(qg)}\,c_{1,1}^{(qg)}(E_T^2/\mu_F^2) + \frac{2}{3\pi}b_0\left(\frac{1}{4}C_F + C_A\right), \quad \text{(B.7)}$$

$$c_{2,2}^{(q\bar{q})}(E_T^2/\mu_F^2, E_T^2/\mu^2) = \frac{1}{2}\left[c_{1,1}^{(q\bar{q})}(E_T^2/\mu_F^2)\right]^2 + c_{1,2}^{(q\bar{q})}\,c_{1,0}^{(q\bar{q})}(E_T^2/\mu_F^2, E_T^2/\mu^2) + \\ + \frac{1}{\pi}b_0\Big[\gamma_E\Big(4C_F - \frac{3}{2}C_A\Big) - (2C_F - C_A)\ln 2 + \frac{1}{2}\pi b_0 +$$

$$
\begin{aligned}
&\quad +\Big(2C_F - \frac{1}{2}C_A\Big)\left(\frac{K}{2\pi b_0} - \ln\frac{2E_T^2}{\mu^2}\right)\Bigg],\\
c_{2,2}^{(qg)}(E_T^2/\mu_F^2, E_T^2/\mu^2) &= \frac{1}{2}\left[c_{1,1}^{(qg)}(E_T^2/\mu_F^2)\right]^2 + c_{1,2}^{(qg)}\, c_{1,0}^{(qg)}(E_T^2/\mu_F^2, E_T^2/\mu^2) + \\
&\quad + \frac{1}{\pi}b_0\Bigg[\gamma_E\Big(\frac{1}{2}C_F + 2C_A\Big) - C_A\ln 2 + \frac{3}{8}C_F + \\
&\quad + \Big(\frac{1}{2}C_F + C_A\Big)\left(\frac{K}{2\pi b_0} - \ln\frac{2E_T^2}{\mu^2}\right)\Bigg], \qquad \text{(B.8)}
\end{aligned}
$$

where the coefficients $c_{1,2}$, $c_{1,1}$, $c_{1,0}$ and K are given in eqs. (B.2), (B.3), (B.4) and (4.10).

Our prediction for the coefficients in eq. (B.5) can be used to check future NNLO calculations of the prompt-photon production cross section. Alternatively, when these calculations become available, they can provide a highly non-trivial check of our NNL resummation.

C. Photoproduction of prompt photons

In hadron-photon collisions the inclusive production of a single prompt photon is due to the process

$$
H_1(P_1) + \gamma(P_2) \to \gamma(p) + X\,. \qquad \text{(C.1)}
$$

We use the same kinematics notation as in the hadroproduction case (cf. sect. 3) and we write the prompt-photon photoproduction cross section integrated over η at fixed E_T as follows:

$$
\frac{d\sigma_\gamma^{(\mathrm{ph})}(x_T, E_T)}{dE_T} = \left(\frac{d\sigma_\gamma^{(\mathrm{ph})}(x_T, E_T)}{dE_T}\right)_{\mathrm{hadronic}} + \left(\frac{d\sigma_\gamma^{(\mathrm{ph})}(x_T, E_T)}{dE_T}\right)_{\mathrm{pointlike}}. \qquad \text{(C.2)}
$$

The hadronic contribution to the cross section is completely analogous to the right-hand side of eq. (3.5) apart from replacing $f_{b/H_2}(x_2, \mu_F^2)$ with the parton distribution $f_{b/\gamma}(x_2, \mu_F^2)$ of incoming photon.

The second contribution on the right-hand side of eq. (C.2) is due to point-like interactions of the incoming photon with high-momentum partons. The point-like cross section can in turn be decomposed in direct and fragmentation components

$$
\begin{aligned}
\left(\frac{d\sigma_\gamma^{(\mathrm{ph})}(x_T, E_T)}{dE_T}\right)_{\mathrm{pointlike}} &= \frac{1}{E_T^3}\sum_a \int_0^1 dx_1\, f_{a/H_1}(x_1, \mu_F^2) \times \\
\times \int_0^1 dx \Bigg\{ \delta\Big(x - \frac{x_T}{\sqrt{x_1}}\Big)\, &\hat\sigma_{a\gamma\to\gamma}(x, \alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2) + \\
+ \sum_c \int_0^1 dz\, z^2\, d_{c/\gamma}(z, \mu_f^2)\, \delta\Big(x - \frac{x_T}{z\sqrt{x_1}}\Big)\, &\hat\sigma_{a\gamma\to c}(x, \alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2)\Bigg\}. \qquad \text{(C.3)}
\end{aligned}
$$

The rescaled partonic cross sections $\hat{\sigma}_{a\gamma\to\gamma}$ and $\hat{\sigma}_{a\gamma\to c}$ have perturbative QCD expansions similar to eqs. (3.6) and (3.7). In particular, for the point-like direct component we have

$$\begin{aligned}
&\hat{\sigma}_{a\gamma\to\gamma}(x, \alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2) = \\
&= \alpha^2 \left[\hat{\sigma}^{(0)}_{a\gamma\to d\gamma}(x) + \sum_{n=1}^{\infty} \alpha_s^n(\mu^2)\, \hat{\sigma}^{(n)}_{a\gamma\to\gamma}(x; E_T^2, \mu^2, \mu_F^2, \mu_f^2) \right] ,
\end{aligned} \tag{C.4}$$

where the only non-vanishing terms at LO are those due to the Compton scattering subprocesses

$$q + \gamma \to q + \gamma \, , \qquad \bar{q} + \gamma \to \bar{q} + \gamma \, , \tag{C.5}$$

whose contribution to the cross section is

$$\hat{\sigma}^{(0)}_{q\gamma\to q\gamma}(x) = \hat{\sigma}^{(0)}_{\bar{q}\gamma\to\bar{q}\gamma}(x) = \pi\, e_q^4 \, \frac{x^2}{\sqrt{1-x^2}} \left(1 + \frac{x^2}{4} \right) . \tag{C.6}$$

To perform soft-gluon resummation at high E_T, we work as usual in N-moment space by defining

$$\sigma^{(\mathrm{ph})}_{\gamma,\,N}(E_T) \equiv \int_0^1 dx_T^2 \, (x_T^2)^{N-1} \, E_T^3 \frac{d\sigma^{(\mathrm{ph})}_{\gamma}(x_T, E_T)}{dE_T} \, . \tag{C.7}$$

The resummation of the large-N corrections to the N-moments of the hadronic contribution in eq. (C.2) is exactly the same as for the hadroproduction case discussed in sect. 4. Moreover, in the large-N limit, the point-like contribution turns out to be dominant: the hadronic contribution involves the additional convolution with the photon parton density $f_{b/\gamma}$ and this implies its suppression by a relative factor of $\mathcal{O}(1/N)$. We can thus limit ourselves to considering the point-like cross section.

In the case of the point-like contribution, one can repeat the argument in sect. 4.1 on the relative size of the fragmentation component and of the various direct subprocesses. Up to NLL accuracy, we then conclude that soft-gluon resummation in the photoproduction cross section (C.7) is controlled by the point-like direct channels $q\gamma \to \gamma$ and $\bar{q}\gamma \to \gamma$. The all-order resummation formulae for the corresponding partonic cross sections are

$$\begin{aligned}
\hat{\sigma}^{(\mathrm{res})}_{q\gamma\to\gamma,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2) &= \alpha^2\, \hat{\sigma}^{(0)}_{q\gamma\to q\gamma,\,N}\, C_{q\gamma\to\gamma}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) \times \\
&\quad \times \Delta_{N+1}^{q\gamma\to q\gamma}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) \, , \qquad (C.8) \\
\hat{\sigma}^{(\mathrm{res})}_{\bar{q}\gamma\to\gamma,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2) &= \hat{\sigma}^{(\mathrm{res})}_{q\gamma\to\gamma,\,N}(\alpha_s(\mu^2); E_T^2, \mu^2, \mu_F^2, \mu_f^2) \, , \qquad (C.9)
\end{aligned}$$

where

$$Q^2 = 2E_T^2 \, , \tag{C.10}$$

and $\hat{\sigma}^{(0)}_{q\gamma\to q\gamma,\,N}$ are the N-moments with respect to x^2 of eq. (C.6)

$$\hat{\sigma}^{(0)}_{q\gamma\to q\gamma,\,N} = \pi\, e_q^2 \, \frac{1}{4} \, \frac{\Gamma(1/2)\,\Gamma(N+1)}{\Gamma(N+5/2)} \, (7+5N) \, . \tag{C.11}$$

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The radiative factor $\Delta_N^{q\gamma\to q\gamma}$ and the N-independent function $C_{q\gamma\to\gamma}$ in eq. (4.2) can directly be related to the analogous contributions $\Delta_N^{qg\to q\gamma}$ and $C_{qg\to\gamma}$ to the qg channel in the hadroproduction process.

The radiative factor $\Delta_N^{q\gamma\to q\gamma}$ is obtained from the factorized expression (4.6) for $\Delta_N^{qg\to q\gamma}$, namely from $\Delta_N^{qg\to q\gamma} = \Delta_N^q \Delta_N^g J_N^q \Delta_N^{(\mathrm{int})\, qg\to q\gamma}$, by switching off soft-gluon radiation from the incoming gluon. This amounts to set $C_A = 0$ in Δ_N^g and $\Delta_N^{(\mathrm{int})\, qg\to q\gamma}$. Using the explicit formulae in eqs. (4.7), (4.9) and (4.14), (4.16), this implies that up to NLL accuracy we can neglect both Δ_N^g and $\Delta_N^{(\mathrm{int})\, qg\to q\gamma}$ and we have the simple result:

$$
\begin{aligned}
&\Delta_N^{q\gamma\to q\gamma}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) = \\
&\quad = \Delta_N^q(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2)\; J_N^q(\alpha_s(\mu^2), Q^2/\mu^2)\,. \qquad \text{(C.12)}
\end{aligned}
$$

Note that no soft-gluon interference factor $\Delta_N^{(\mathrm{int})}$ appears in eq. (C.12). Prompt-photon photoproduction at threshold is dominated by an underlying hard-scattering that involves only two hard partons and then, in agreement with the general discussion in sect. 2, soft-gluon interferences have to cancel.

The explicit NLL expansion of eq. (C.12) gives

$$
\begin{aligned}
\Delta_N^{q\gamma\to q\gamma}\left(\alpha_s(\mu^2), \frac{Q^2}{\mu^2}; \frac{Q^2}{\mu_F^2}\right) &= \exp\Big\{ \ln N\, g_{q\gamma}^{(1)}(b_0\alpha_s(\mu^2)\ln N) + \\
&+ g_{q\gamma}^{(2)}(b_0\alpha_s(\mu^2)\ln N, Q^2/\mu^2; Q^2/\mu_F^2) + \mathcal{O}(\alpha_s(\alpha_s \ln N)^k)\Big\}\,, \qquad \text{(C.13)}
\end{aligned}
$$

where the LL and NLL terms $g^{(1)}$ and $g^{(2)}$ are expressed in terms of the auxiliary functions $h^{(1)}$ and $h^{(2)}$ of eqs. (4.24) and (4.25):

$$
g_{q\gamma}^{(1)}(\lambda) = C_F\, h^{(1)}(\lambda/2)\,, \qquad \text{(C.14)}
$$

$$
\begin{aligned}
g_{q\gamma}^{(2)}\left(\lambda, \frac{Q^2}{\mu^2}; \frac{Q^2}{\mu_F^2}\right) &= 2\, C_F\, h^{(2)}(\lambda/2) + \frac{4C_F\,\gamma_E - 3C_F}{4\pi b_0}\ln(1-\lambda) - \\
&- \frac{C_F}{\pi b_0}\lambda \ln\frac{Q^2}{\mu_F^2} + \frac{C_F}{\pi b_0}\Big[\lambda + \ln(1-\lambda)\Big]\,\ln\frac{Q^2}{\mu^2}\,. \qquad \text{(C.15)}
\end{aligned}
$$

The N-independent function $C_{q\gamma\to\gamma}(\alpha_s)$ has the following perturbative expansion:

$$
\begin{aligned}
C_{q\gamma\to\gamma}(\alpha_s(\mu^2), Q^2/\mu^2; Q^2/\mu_F^2) &= 1 + \frac{\alpha_s(\mu^2)}{\pi}\, C_{q\gamma\to\gamma}^{(1)}(Q^2/\mu_F^2) + \\
&+ \sum_{n=2}^{+\infty}\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n C_{q\gamma\to\gamma}^{(n)}(Q^2/\mu^2; Q^2/\mu_F^2)\,. \qquad \text{(C.16)}
\end{aligned}
$$

Note that the first-order coefficient $C_{q\gamma\to\gamma}^{(1)}$ does not depend on the renormalization scale. Its explicit expression is obtained from that of $C_{qg\to\gamma}^{(1)}$ by setting $C_A = 0$ and $b_0 = 0$ in eq. (4.34):

$$
\begin{aligned}
C_{q\gamma\to\gamma}^{(1)}(Q^2/\mu_F^2) = C_F\, \Big\{ &\frac{1}{2}\gamma_E^2 + \frac{3}{4}\gamma_E - \frac{1}{10}\ln 2 - \frac{1}{2}\frac{K_q}{C_F} + \\
&+ \frac{\pi^2}{30} + \frac{1}{2}\ln^2 2 - \Big(\gamma_E - \frac{3}{4}\Big)\ln\frac{Q^2}{\mu_F^2}\Big\}\,. \qquad \text{(C.17)}
\end{aligned}
$$

Expanding the resummation formula (C.8) in powers of α_s we can derive the large-N behaviour of the NLO and NNLO cross sections $\hat{\sigma}^{(1)}_{q\gamma\to\gamma}$ and $\hat{\sigma}^{(2)}_{q\gamma\to\gamma}$ of eq. (C.4).

At NLO we find

$$\hat{\sigma}^{(1)}_{q\gamma\to\gamma,\,N}(E_T^2,\mu^2,\mu_F^2,\mu_f^2) = \hat{\sigma}^{(0)}_{q\gamma\to q\gamma,\,N}\ \Big[\ c_{1,2}^{(q\gamma)}\ \ln^2 N + c_{1,1}^{(q\gamma)}(E_T^2/\mu_F^2)\ln N + \\ + c_{1,0}^{(q\gamma)}(E_T^2/\mu_F^2) + \mathcal{O}(1/N)\Big]\,, \tag{C.18}$$

where

$$\begin{aligned} c_{1,2}^{(q\gamma)} &= \frac{1}{2\pi}C_F\,, \\ c_{1,1}^{(q\gamma)}(E_T^2/\mu_F^2) &= \frac{1}{\pi}C_F\left(\gamma_E + \frac{3}{4} - \ln\frac{2E_T^2}{\mu_F^2}\right)\,, \\ c_{1,0}^{(q\gamma)}(E_T^2/\mu_F^2) &= \frac{1}{\pi}C^{(1)}_{q\gamma\to\gamma}(2E_T^2/\mu_F^2)\,. \end{aligned} \tag{C.19}$$

This result agrees with the large-N limit of the NLO analytic expressions computed in ref. [48].

At NNLO we predict

$$\hat{\sigma}^{(2)}_{q\gamma\to\gamma,\,N}(E_T^2,\mu^2,\mu_F^2,\mu_f^2) = \hat{\sigma}^{(0)}_{q\gamma\to q\gamma,\,N}\Big[c_{2,4}^{(q\gamma)}\ \ln^4 N + c_{2,3}^{(q\gamma)}(E_T^2/\mu_F^2)\ln^3 N + \\ + c_{2,2}^{(q\gamma)}(E_T^2/\mu_F^2,\,E_T^2/\mu^2)\ln^2 N + \mathcal{O}(\ln N)\Big]\,, \tag{C.20}$$

where

$$c_{2,4}^{(q\gamma)} = \frac{1}{2}\ \left[c_{1,2}^{(q\gamma)}\right]^2 = \frac{1}{8\pi^2}C_F\,, \tag{C.21}$$

$$\begin{aligned} c_{2,3}^{(q\gamma)}(E_T^2/\mu_F^2) &= c_{1,2}^{(q\gamma)}\ c_{1,1}^{(q\gamma)}(E_T^2/\mu_F^2) + \frac{1}{6\pi}C_F b_0 \\ &= \frac{1}{2\pi^2}C_F\left[C_F\left(\gamma_E + \frac{3}{4} - \ln\frac{2E_T^2}{\mu_F^2}\right) + \frac{\pi}{3}b_0\right]\,, \end{aligned} \tag{C.22}$$

$$\begin{aligned} c_{2,2}^{(q\gamma)}\left(\frac{E_T^2}{\mu_F^2},\frac{E_T^2}{\mu^2}\right) &= \frac{1}{2}\ \left[c_{1,1}^{(q\gamma)}(E_T^2/\mu_F^2)\right]^2 + c_{1,2}^{(q\gamma)}\ c_{1,0}^{(q\gamma)}(E_T^2/\mu_F^2) + \\ &\quad + \frac{C_F b_0}{2\pi}\left(\gamma_E + \frac{3}{4} + \frac{K}{2\pi b_0} - \ln\frac{2E_T^2}{\mu^2}\right) = \\ &= \frac{C_F}{2\pi^2}\left[C^{(1)}_{q\gamma\to\gamma}(2E_T^2/\mu_F^2) + \Big(C_F + \pi b_0\Big)\left(\gamma_E + \frac{3}{4} - \ln\frac{2E_T^2}{\mu^2}\right) + \frac{1}{2}K\right]\,, \end{aligned} \tag{C.23}$$

and the coefficient K is given in eqs. (4.10).

References

[1] P. Aurenche, R. Baier, A. Douiri, M. Fontannaz and D. Schiff, *Phys. Lett.* **B 140** (1984) 87;
P. Aurenche, R. Baier, M. Fontannaz and D. Schiff, *Nucl. Phys.* **B 297** (1988) 661.

[2] H. Baer, J. Ohnemus and J.F. Owens, *Phys. Rev.* **D 42** (1990) 61;
P. Aurenche, R. Baier and M. Fontannaz, *Phys. Rev.* **D 42** (1990) 1440.

[3] L.E. Gordon and W. Vogelsang, *Phys. Rev.* **D 48** (1993) 3136.

[4] E.L. Berger and J. Qiu, *Phys. Lett.* **B 248** (1990) 371, *Phys. Rev.* **D 44** (1991) 2002.

[5] E.L. Berger, X. Guo and J. Qiu, *Phys. Rev. Lett.* **76** (1996) 2234 [hep-ph/9512281]; *Phys. Rev.* **D 54** (1996) 5470 [hep-ph/9605324, hep-ph/9708408], also published in *Proc. of the 32nd Rencontres de Moriond: QCD and High-Energy Hadronic Interactions*, J. Tran Than Van ed., Editions Frontières, Paris 1997, p. 267.

[6] P. Aurenche, M. Fontannaz, J.Ph. Guillet, A. Kotikov and E. Pilon, *Phys. Rev.* **D 55** (1997) R1124 [hep-ph/9606287].

[7] S. Catani, M. Fontannaz and E. Pilon, hep-ph/9803475, preprint CERN-TH-98-82.

[8] B. Bailey, J. Ohnemus and J.F. Owens, *Phys. Rev.* **D 46** (1992) 2018.

[9] L.E. Gordon and W. Vogelsang, *Phys. Rev.* **D 50** (1994) 1901.

[10] F. Aversa, P. Chiappetta, M. Greco and J.Ph. Guillet, *Nucl. Phys.* **B 327** (1989) 105.

[11] P. Aurenche, R. Baier, M. Fontannaz, J.F. Owens and M. Werlen, *Phys. Rev.* **D 39** (1989) 3275.

[12] M. Gluck, L.E. Gordon, E. Reya and W. Vogelsang, *Phys. Rev. Lett.* **73** (1994) 388.

[13] J. Huston et al., *Phys. Rev.* **D 51** (1995) 6139 [hep-ph/9501230].

[14] W. Vogelsang and A. Vogt, *Nucl. Phys.* **B 453** (1995) 334 [hep-ph/9505404].

[15] C. De Marzo et al., NA24 Collaboration, *Phys. Rev.* **D 36** (1987) 8;
M. Bonesini et al., WA70 Collaboration, *Z. Physik* **C 38** (1988) 371;
A. Bernasconi et al., UA6 Collaboration, *Phys. Lett.* **B 206** (1988) 163.

[16] G. Alverson et al., E706 Collaboration, *Phys. Rev.* **D 48** (1993) 5.

[17] A.L.S. Angelis et al., CMOR Collaboration, *Nucl. Phys.* **B 327** (1989) 541;
E. Anassontzis, R806 Collaboration, *Z. Physik* **C 13** (1982) 277;
T. Akesson et al., R806 Colaboration, *Sov. J. Nucl. Phys.* **51** (1990) 836.

[18] C. Albajar et al., UA1 Collaboration, *Phys. Lett.* **B 209** (1988) 385;
J. Alitti et al., UA2 Collaboration, *Phys. Lett.* **B 263** (1991) 544.

[19] F. Abe et al., CDF Collaboration, *Phys. Rev. Lett.* **73** (1994) 2662;
S. Abachi et al., D0 Collaboration, *Phys. Rev. Lett.* **77** (1996) 5011 [hep-ex/9603006].

[20] S. Frixione, M.L. Mangano, P. Nason and G. Ridolfi, *Nucl. Phys.* **B 431** (1994) 453, *ibid.* **405** (1993) 507, hep-ph/9702287, preprint CERN-TH-97-16, to be published in Heavy Flavours II, A.J. Buras and M. Lindner eds., World Scientific.

[21] H. Baer, M.H. Reno, *Phys. Rev.* **D 54** (1996) 2017 [hep-ph/9603209].

[22] W. Vogelsang, private communication.

[23] L. Apanasevich et al., hep-ex/9711017, E706 Collaboration, preprint FERMILAB–Pub–97/351-E.

[24] A.D. Martin, R.G. Roberts, W.J. Stirling and R.S. Thorne, *Eur. Phys. J.* **C4** (1998) 4 [hep-ph/9803445].

[25] M. Fontannaz, in Proc. of the *32nd Rencontres de Moriond: QCD and High-Energy Hadronic Interactions*, ed. J. Tran Than Van, Editions Frontières, Paris, 1997, p. 235.

[26] F. Abe et al., CDF Collaboration, *Phys. Rev. Lett.* **77** (1996) 438 [hep-ex/9601008].

[27] R. Bonciani, University of Florence Thesis, July 1997 (in Italian).

[28] R. Bonciani, S. Catani, M.L. Mangano and P. Nason, hep-ph/9801375, preprint CERN-TH-98-31 to appear in *Nucl. Phys.* **B**.

[29] R. Bonciani, S. Catani, M.L. Mangano and P. Nason, in preparation.

[30] E. Laenen, G. Oderda and G. Sterman, hep-ph/9806467.

[31] G. Sterman, *Nucl. Phys.* **B 281** (1987) 310.

[32] S. Catani and L. Trentadue, *Nucl. Phys.* **B 327** (1989) 323.

[33] S. Catani and L. Trentadue, *Nucl. Phys.* **B 353** (1991) 183.

[34] See: G. Sterman, in Proc. *10th Topical Workshop on Proton-Antiproton Collider Physics*, R. Raja and J. Yoh eds., AIP Press, New York 1996, p. 608;
S. Catani, in Proc. of the *32nd Rencontres de Moriond: QCD and High-Energy Hadronic Interactions*, ed. J. Tran Than Van, Editions Frontières, Paris 1997, p. 331 [hep-ph/9709503]; and references therein.

[35] A. Bassetto, M. Ciafaloni and G. Marchesini, *Phys. Rep.* **100** (1983) 201;
Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller and S.I. Troyan, *Basics of Perturbative QCD*, Editions Frontières, Gif-sur-Yvette 1991.

[36] N. Kidonakis and G. Sterman, *Phys. Lett.* **B 387** (1996) 867; *Nucl. Phys.* **B 505** (1997) 321 [hep-ph/9705234].

[37] N. Kidonakis, G. Oderda and G. Sterman, *Nucl. Phys.* **B 525** (1998) 299 [hep-ph/9801268].

[38] N. Kidonakis, G. Oderda and G. Sterman, hep-ph/9803241, preprint ITP-SB-98-23.

[39] J. Kodaira and L. Trentadue, *Phys. Lett.* **B 112** (1982) 66.

[40] S. Catani, E. d'Emilio and L. Trentadue, *Phys. Lett.* **B 211** (1988) 335.

[41] S. Catani, M.L. Mangano, P. Nason and L. Trentadue, *Phys. Lett.* **B 378** (1996) 329 [hep-ph/9602208]; *Nucl. Phys.* **B 478** (1996) 273 [hep-ph/960435].

[42] A.P. Contogouris, N. Mebarki and S. Papadopoulos, *Int. J. Mod. Phys.* **A 5** (1990) 1951;
A.P. Contogouris and S. Papadopoulos, *Mod. Phys. Lett.* **A 5** (1990) 901.

[43] S. Catani, L. Trentadue, G. Turnock and B.R. Webber, *Nucl. Phys.* **B 407** (1993) 3.

[44] S. Catani, G. Marchesini and B.R. Webber, *Nucl. Phys.* **B 349** (1991) 635.

[45] M. Krämer, E. Laenen and M. Spira, *Nucl. Phys.* **B 511** (1998) 523 [hep-ph/9611272].

[46] H. Contopanagos, E. Laenen and G. Sterman, *Nucl. Phys.* **B 484** (1997) 303 [hep-ph/9604313].

[47] S. Catani, G. Turnock, B.R. Webber and L. Trentadue, *Phys. Lett.* **B 263** (1991) 491.

[48] P. Aurenche, A. Douiri, R. Baier, M. Fontannaz and D. Schiff, *Z. Physik* **C 24** (1984) 309.